# What criteria should be used for redistricting reform?

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## Abstract

Congressional redistricting plans for Pennsylvania, with an emphasis on the newly enacted 2018 plan, have been evaluated for fairness and responsiveness to voters. This and other submitted plans that adhered to the traditional reform criteria of compactness and not splitting political boundaries have half as much bias favoring Republicans as the unconstitutional map of 2011. For fairer maps, it appears to be necessary to "anti-gerrymander" by relaxing the traditional criteria in order to overcome the political geography in Pennsylvania which apparently makes a Democratic gerrymander practically impossible. The methodology uses five statewide data bases at the precinct level and suitably constructed seats/votes curves. If fairness and responsiveness are valued more than political geography, then they should be made explicit criteria in congressional districting, at least in Pennsylvania.

## 1. Introduction

If redistricting is done well, there would be no elections using a flawed map and no need for subsequent lawsuits to overturn such a map. The focus of this paper is on the criteria that should be used by a redistricting commission rather than on criteria for challenging an approved map in the courts. In other words, how should the map be drawn in the first place rather than how could a bad map be overturned.[1] This redirects the more

[1] Although there have been notable decisions by some appellate courts regarding redistricting, courts on the whole have had difficulty engaging the issue effectively, at least in part because it can be

common emphasis of election law from the courts to the legislature.

The criteria of equal population, contiguity and voting rights considerations are required by law and will be employed throughout this paper. Beyond those, traditional redistricting uses the so-called neutral criteria of compactness and not splitting political boundaries, such as counties and municipalities. These additional criteria are written into some states' constitutions or legislation, but not uniformly.[2] While these traditional criteria,[3] if actually adhered to, would prohibit the worst abuses of partisan gerrymandering, they do not necessarily prevent unintentional gerrymandering which comes about from political geography as was shown for Florida (Chen and Rodden, 2013).[4] This paper does an up to date analysis of congressional redistricting in Pennsylvania to determine whether unintentional gerrymandering also is likely to occur in this state. In Sections 2 and 3 it is shown that all the many traditional maps that have recently been drawn for the Pennsylvania (PA) congressional delegation would have resulted

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seen as legislative law, not judicial law. Cover (2018) is one of the many articles that review court cases.

[2] For example, in Pennsylvania these additional criteria are in Article II, section 16 of the constitution for districting of the state legislature, but congressional districting is not in the PA constitution.

[3] There are additional traditional criteria, see, e.g. Hirsch and Ortiz (2005) but this paper will focus on compactness and not splitting political boundaries, as these are the only ones in the PA constitution and the ones required by the Supreme Court of PA (abbr. SCOPA).

[4] An obvious aspect of political geography is that Democrats disproportionally live in cities.

in unintentional gerrymandering. Therefore, this paper advocates a different approach to redistricting in PA.

The premise of the policy advocated in this paper is that redistricting should be fair and responsive to voters. Fairness to voters means that like-minded voters with one general viewpoint should be equally empowered as like-minded voters of a different general viewpoint (Nagle, 2017).[5] Responsiveness to voters means that a state's representation in a legislative body responds to changes in voters' preferences. Responsiveness is often described as having districts that are competitive.

If fairness and responsiveness are valued more than political geography, then a redistricting commission should be tasked with choosing a map that has the least bias and substantial responsiveness[6] from among the many that the commission could draw and that citizen map drawers might submit to it.[7] Of course, this requires quantitative methods to estimate bias and responsiveness. Several methods will be used in this paper.

All methods have to use past election results at the precinct level to serve as mock elections for a redistricting commission to test maps before any election. Remarkably, many reformers prefer to ban the use of past election results. This is understandable if politicians are the commissioners, as they can

[5] In America the different viewpoints are usually thought of as Democratic and Republican, although it is perhaps better to think of them as progressive and conservative, especially because the latter distinction includes independents and minor parties.

[6] Whereas zero bias is the obvious ideal value, the ideal amount of responsiveness is a more difficult issue (McGann et al., 2016, p.67; Nagle, 2017).

[7] Courts have the threshold bias problem of having to decide how much is too much. That problem is alleviated for redistricting commissions that would only have to try to minimize bias, subject, of course, with balancing with the traditional and any other criteria.

use such data to aggressively gerrymander for partisan advantage. However, if there is an independent commission, the use of past election results would allow it to bring about greater fairness and responsiveness.[8]

The estimation method used in this paper is based on the well-known seats/votes concept. The Seats/Votes graph (*abbr.* S/V) is a powerful way to evaluate the fairness and responsiveness of a districting plan, as has been long recognized in the political science literature.[9] Specifics about how S/V graphs are drawn for the purpose of evaluating maps are described in section 2.[10]

The S/V curve immediately reveals intuitively appealing quantities to evaluate bias, such as (i) the fraction of seats at 50% of the vote and (ii) the vote required to obtain half the seats; furthermore, it reveals responsiveness.[11] Other ways to evaluate bias include the efficiency gap (McGhee, 2014) and a

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[8] However, ingrained notions die hard; the amendment to the PA constitution (SB22) proposed by reform groups in PA banned the use of past election results even for the truly independent commission that they proposed.

[9] Complaints about S/V graphs are that they are too complicated for courts to understand and that they are counterfactuals (Stephanopoulos and McGhee, 2015). Although it is well understood in the social sciences that counterfactuals are essentially estimates of events that have not occurred, semantically, the word subconsciously connotes that it is something that is contrary to fact. It is hard to imagine planning in any context, science or engineering or social science, that does not use “counterfactuals”. See also McGann et al. (2016), p. 221 for defending the use of counterfactuals.

[10] The S/V graphs in this paper are more appropriate than those proposed earlier by this author (Nagle,2015).

[11] Although responsiveness is often reported when the statewide vote is 50%, for states with a dominant party like Maryland, the more appropriate measure of responsiveness should be evaluated at the mean statewide fraction. Likewise, bias may be better measured at the same mean statewide vote using a symmetrical counterfactual as will be discussed in Appendix B.

new measure introduced recently in this journal (Warrington, 2017). Appendix A explains why this paper does not use those measures. Appendix B shows that the critical test developed in Appendix A can be met by the S/V curves used in Section 2.

Section 2 gives results for the new congressional map adopted by the Supreme Court of Pennsylvania, to be abbreviated the SCOPA map. Although the SCOPA plan is much better than the previous highly gerrymandered plan,[12] it is shown that it still is biased in favor of conservative voters. Appendix C remarks on an interesting anomaly with respect to the SCOPA map and the 2016 presidential election.

When SCOPA declared the 2011 Pennsylvania congressional plan unconstitutional, it tasked the legislature with drawing a new plan that adhered to the traditional, so-called neutral, criteria. Before the allotted deadline, missed by the legislature, many *amici* plans were submitted before SCOPA imposed its plan drawn by its special master. Section 3 analyzes these *amici* plans as prime examples of plans that were shown to clearly adhere to the traditional criteria. These plans are just as biased as the SCOPA plan, suggesting that the traditional criteria will generally lead to unintentional gerrymanders in PA. In contrast, section 4 presents plans that do not conform to the traditional criteria and that are almost fair. As discussed in Section 5 this means that reform bills in PA that have eschewed fairness and responsiveness in favor of adhering to the traditional criteria would likely not accomplish what many reformers and the public want. Section 6 suggests

[12] The previous 2011 map was struck down in 2018 as violating the Free and Equal Elections Clause of the PA constitution. Full documentation of the lawsuit is available from the Brennan Center for Justice http://www.brennancenter.org/legal-work/league-women-voters-v-pennslyvania.

language that might be used in future reform laws to promote fairness and responsiveness to voters.

## 2. Seats/Votes (S/V) graphs for the SCOPA map

A seats/votes graph traditionally has the form of the number of a party's seats S for any percentage V of that party's statewide vote.[13] This general form could be used to predict the outcome of a specific election, but a seats/votes graph to evaluate a districting plan is different from that. To predict a specific election, one would consider the incumbency advantage (Gelman and King, 1990) and the relative popularity of the candidates. For example, if one party had more incumbents in the competitive districts, its S/V graph would predict more seats for a given statewide vote than if that party had fewer incumbents. This and other factors, such as the amount of campaign expenditure, are extrinsic to the plan itself. In order to remove these extrinsic factors and to approximate intrinsic partisan preference of the plan, the political science literature has long recognized the importance of using the results of statewide elections to construct S/V graphs.[14] Of course, one still expects different S/V graphs for a map upon using different election results just as different results are obtained from different actual elections. Having several past

[13] When there are only two dominant parties, as in all the examples in this paper, the vote V will be the percentage of the two-party vote. Importantly, this party centric definition can be generalized to progressive versus conservative voters if those attributes are not well aligned with the parties.

[14] A few examples are: Backstrom et al. (1990) advocated using election returns for a low-profile statewide base race, Gronke & Wilson (1999) averaged three races, McDonald (2014) used presidential votes, and Best et al. (2018) used nine statewide races in their analysis of North Carolina and ten for Iowa.

elections allows one to estimate the uncertainty in the estimate of the intrinsic S/V curve.

The S/V examples in this paper are for Pennsylvania congressional districting. Five different data bases of past PA election results are used for each of the over 9000 voting districts (precincts) in order to evaluate the SCOPA plan that had not yet had an election as of the original writing of this paper, as well as plans that will never be enacted. One data base is the Cook partisan voter index (PVI) which consists of the two presidential elections of 2012 and 2016. The second data base, designated 7s, is the aggregated votes for the statewide elections of 2012 (president, senator, attorney general, auditor general and treasurer) and for 2014 (governor) plus registration. The last three data bases are the separate election results for 2016 for president, for senate, and for the row offices (aggregated votes for attorney general, auditor general and treasurer).[15]

Given the result of a past election at the precinct level, it is straightforward to add those votes for the precincts in a map's districts to obtain a partisan preference for each district. One might suppose that this would suffice to predict the number of A seats for the election being applied simply by counting the number of seats with more party A voters than party B voters. However, as is well known (McGann, 2018, pp. 58-59) simple examples show that this naïve counting is deficient.

[15] Apropos of footnote 4, there is no indication that any of these past PA elections involved Democratic candidates who would be considered more conservative than their Republican opponents or *vice versa*. If that were not the case for a particular election, a simple expedient would be not to use such an election or even to count it in the reverse fashion to align it with progressive versus conservative like-minded voters.

Suppose that there are two districts that each have only a few more voters favoring party A than favor party B, so both districts would be counted as A seats according to naïve counting. Contrarily, to take into account fluctuations in voter turnout and random decisions of fence-sitting voters, each district should be counted as only slightly more than half a seat for party A. In this paper, each district is assigned a fraction of an A seat using the probability function shown in Fig. 1. The sum of these fractions for all districts then gives the seat fraction S for the vote fraction V of the applied election.

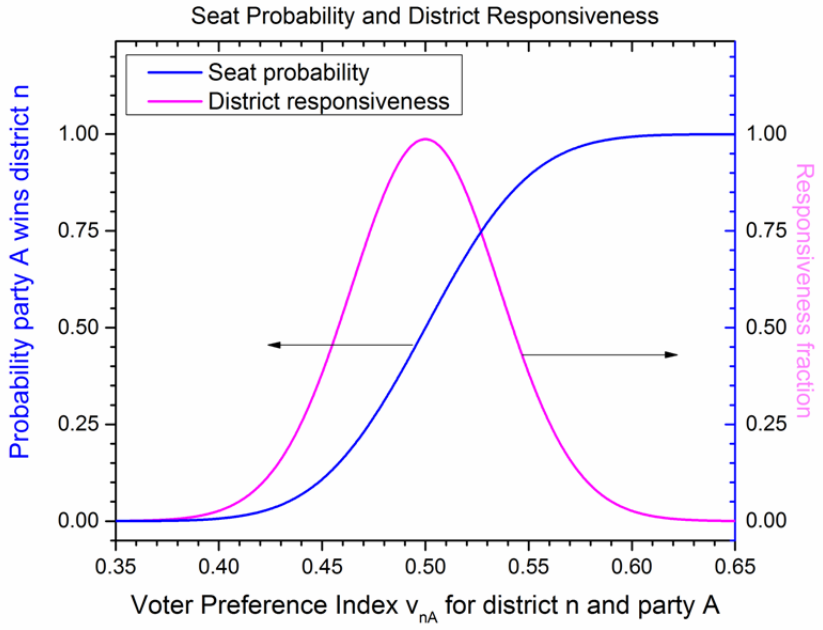



Figure. 1. The blue curve labelled party A seat probability gives the fraction of a seat assigned to party A versus party B's district preference (past vote fraction).[16] The magenta curve gives a measure of the responsiveness of a district.

Thereby, each past election result provides an estimated number of seats for the statewide vote for that election. Figure

[16] Party seat probability P(V) = 1-0.5*(1+prob((V-0.5)/0.04)) where prob is the usual probit function, here with variance 0.04. Results are fairly insensitive in the variance range of 0.04±0.02, similar to the 5% range often mentioned (McGann, 2016, p.59). Responsiveness is quantitatively defined as $R(V) = 1-4*(P(V)-0.5)^2$. More complex methods assign seat probabilities using a logit transformational model applied to past election results (Chen & Cottrell, 2016; Warrington, 2017) or Bayesian methods (Baas & McAuliffe, 2018).

2 shows five results for the SCOPA map. The locations of the letters on Figure 2 show the resulting S versus V where the V is the actual statewide vote for the five data bases used in this paper. For example, letter A is located at the two-party 2016 presidential D vote of 49.62%. Point D shows the anti-majoritarian result in fewer than half D seats for more than half D vote and point C would require nearly 53% D vote for slightly less than half the seats. Points A, B and E are also biased, but less obviously so without the subsequent analysis. The letters A-E in Fig. 2 do indicate the general trend of the true intrinsic seats/votes (S/V) curve of the SCOPA plan. But while the B,C and E points suggest a smooth curve, points A and D would deviate.

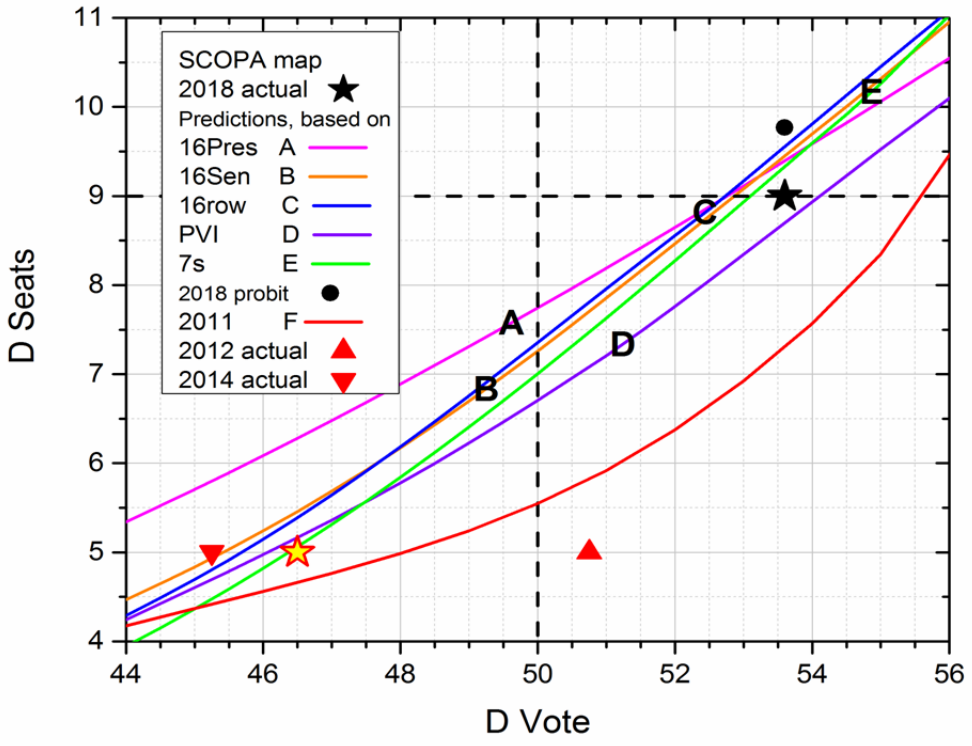



Figure 2. Number of D seats versus D vote for the SCOPA map for five past election data bases of Pennsylvania's 18 congressional districts. The letters A-E locate the statewide D vote for each "election". The continuous curves result from

applying proportional shifts to the vote. The solid star is the result of the 2018 election.[17]

Importantly, each of the lettered points in Fig. 2 can be extended into a full S/V curve for all statewide votes V. These S/V curves, one for each of the five data bases, are shown in Fig. 2 by each of the five curves drawn through each lettered point. Traditionally, such S/V curves have been drawn by shifting the preference of each district the same amount as the statewide vote shifts. This uniform shift assumption is quite unrealistic on its face because it assumes that the same number of D voters would shift in districts with few D's as in districts with many D's.[18] This paper employs an improvement, to be called the proportional shift, which shifts the same proportion of D's (or R's if the shift is to larger statewide D vote) in each district (Nagle, 2015). However, for the important range of statewide vote portrayed in Fig. 2, it makes little difference whether one uses the uniform shift or the proportional shift. Each S/V curve for each past election gives the number of seats for any statewide vote. A comparison of the S/V curves in Fig. 2 indicates that the 2016 presidential election would result in more D seats than the others when the statewide vote is less than 52% D, whereas the PVI data would result in fewer D seats than the others when the statewide vote

[17] It may be noted that the 2018 election occurred after this paper was submitted with this same figure except for the 2018 results. The solid circle assigns more seats to the Democrats using the probit function in footnote 16 to correct for the three closest contests occurring in Republican won districts. Although these S/V curves are not necessarily predictors of actual results due to possible varying strength in the two fields of candidates, any such fluctuations in this election apparently averaged out thereby giving good agreement.

[18] The uniform shift construction of S/V curves has been appropriately criticized (King, 1989).

is greater than 48% D.[19] Appendix C presents evidence that the 2016 presidential election in PA was an anomaly when compared to the other PA elections that year.

Every S/V curve in Fig. 2 indicates bias against voters who voted Democratic in the five data bases from past elections. One simple measure of this bias is the number of R seats minus the number of D seats when the two-party vote is evenly split at 50%. Averaging the five data bases gives a difference of 3.5 seats. Often, half the difference in seats divided by the total number of seats is used - that number is 9.4%. Let us call this the S/V seats measure of bias. Another simple measure of bias, which will be called the S/V votes measure, is the percentage deviation from 50% of the vote needed to obtain half the seats.[20] That is 3% based on 53% vote needed in Fig. 2, disregarding the larger 54% vote needed for the PVI data base. The seats measure and the votes measure only have the same value of bias when the responsiveness is unity as in proportional representation.[21]

Either the seats or votes measure of bias suffices to show that S/V curves for the SCOPA map violate the symmetry criterion for a fair map, namely, that a fair map should be symmetric in the sense that, if party A obtains S seats when it receives V

[19] I am less confident in the PVI data base which I only accessed through Dave's redistricting app (Bradlee, 2010), unlike the other data bases that I acquired from PA department of state. It is surprising that it uniformly gives a smaller number of seats than the 2016 presidential election when it is based upon the average of the 2012 and 2016 presidential elections.

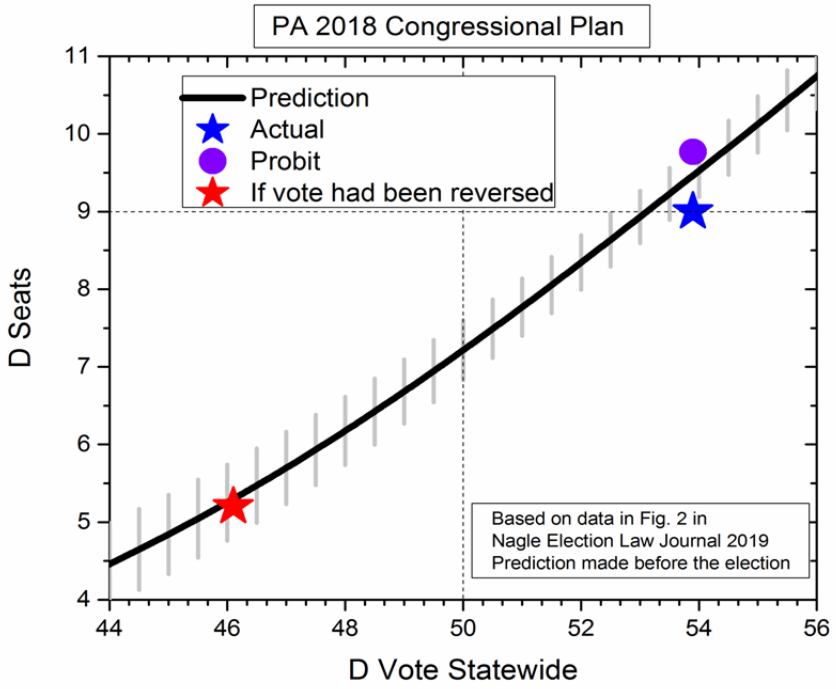
[20] This votes measure is basically an extension of the mean-median measure (McDonald and Best, 2015). This extension, by using the S/V graph, avoids over-emphasis on the median district.

[21] Indeed, the ratio of the seats measure (9.4%) to the votes measure (3%) provides the value 3.1 for the responsiveness for the SCOPA map.

vote, then so should party B obtain S seats when it receives V vote.[22] (Grofman and King, 2007, McGann et al., 2016)

The full S/V curves in Fig. 2 also provide a measure of responsiveness, designated by the symbol R, and defined as the increase in the fraction of seats divided by the increase in the fraction of the vote. That is simply obtained from the slope of the curves.[23]

Figure 2 is rather busy with many curves. The next figure, not in the original manuscript, combines the curves and this also gives statistical uncertainties.




[22] A geometric measure has been proposed that quantifies the amount of asymmetry (Nagle, 2015). That measure accounts for both the bias in the seat direction and the bias in the vote direction; this effectively removes responsiveness, and it give values of bias between those of the seats and votes measures when suitably normalized. However, it is not readily intuitively graspable and the sign of the bias becomes ambiguous for some S/V curves that are nearly, but not quite, symmetric, so it will not be featured in this paper.

[23] The slope is the mathematical derivative which varies somewhat with the vote; values of R presented in this paper will be for V = 50%.

## 3. Analysis of Other PA Congressional Maps Conforming to Traditional Criteria

Between the time when SCOPA struck down the 2011 PA congressional map and the deadline that it set for adoption of a new map, many *amici* maps were entered into the court record[24] and other maps were also drawn by individuals. These maps adhered to the Court's directive that maps had to satisfy the traditional criteria of compactness and minimizing splitting political subdivisions, especially counties and municipalities, in addition to the legal criteria of contiguity, equal population,[25] and the federal Voting Rights Act of 1965. Using the 7s past election results data base, Figure 3 shows the average S/V curve of five maps drawn by *amici* and two maps drawn by individuals. It also shows the Republican leaders' plan; it is the most biased in favor of Republicans with only 6 D seats with 50% of the vote.[26] Grofman has referred to such plans as stealth gerrymanders.

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[24] See footnote 11 for the court record.

[25] These maps generally adhered to the federal requirement, critically reviewed by Hirsch and Ortiz (2005), that population deviations be limited to one person in districts consisting of 705,688 people. This meant that each map had to have a minimum of 17 county splits, one less than the number of districts. With the precincts that had been established in 2011, this also meant there was usually one or more split precincts adjacent to each county split. It was not possible to achieve this degree of accuracy in my redrawing of the maps for analysis. My population deviations were usually at the level of one precinct, typically less than 3000 people or less than 0.5% population deviation. These small deviations make less than 0.1% difference in the numerical results for bias and responsiveness.

[26] The Republican leaders' plan was not voted on by the legislature before it was submitted to the governor who rejected it after the report of his appointed expert that faulted it for not sufficiently adhering to the traditional criteria.

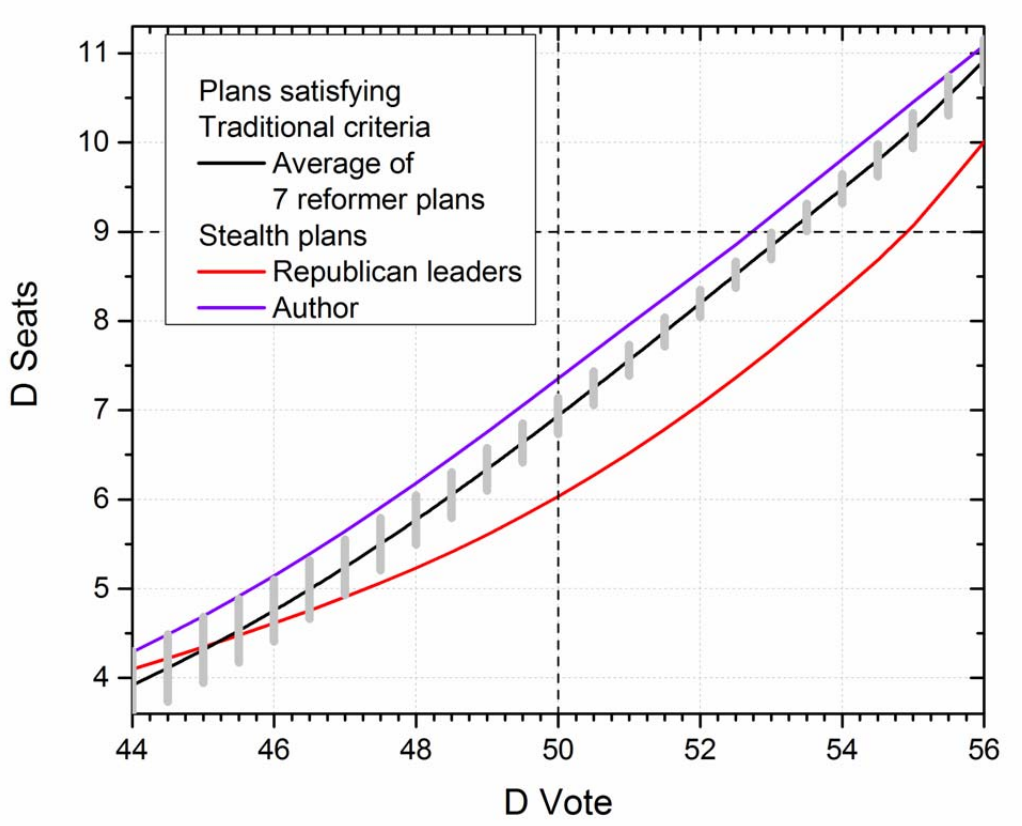


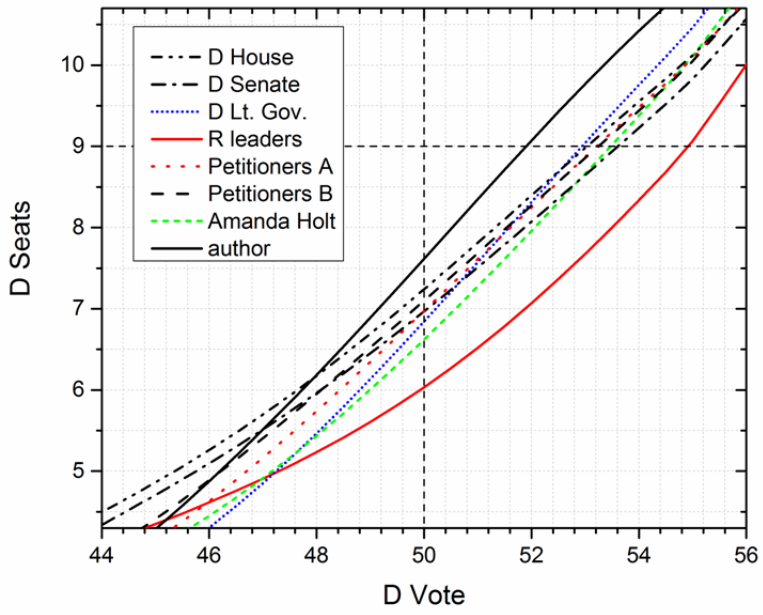

Figure 3. S/V curves for PA congressional redistricting plans that adhere to the traditional criteria. The average is for three maps submitted by Democrats, two maps submitted by the petitioners, one map drawn by Amanda Holt[27], one by a private citizen and one by the author. The Republican leaders in state house and senate submitted a single map. The election data used was the 7s data base.

The plan least biased against Democrats (7.3 D seats at 50% of the vote) was drawn by this author, actively using the 7s data

[27] The author thanks Amanda Holt for sharing several of her maps from her website (http://amandae.com) in Dave's Redistricting App http://gardow.com/davebradlee/redistricting/launchapp.html format. Other maps were hand drawn in that format from the posted images. While there are undoubtedly a few mistakes in precinct assignment, most of the districts follow easily discernible county lines, so errors are insignificant. Images of the maps mentioned in this paper may be viewed at http://lipid.phys.cmu.edu/nagle.

base to produce the most responsive and least biased map while adhering to the traditional criteria.[28] The other S/V curves lie between these two S/V curves except at small D vote. Their partisan preferences range from 6.6 to 7.2 D seats at 50% of the vote and 52.9 to 53.6% D vote for half the seats. Although the differences in bias of these seven maps are relatively small, it might be noted that maps drawn by the Democrats and the petitioners are a bit less favorable to Republicans at 50% of the vote than the Holt plan and the Lt. Governor's plan[29]. If one supposes that these two better represent non-partisan map drawing, then it would appear from Fig. 3 that traditional, partisan blind, neutral map drawing would result in a preference of somewhat fewer than 7 D congressional seats in PA for 50% two party D vote.

Colorized Fig. 3 in published paper better shows some of the points made in the last paragraph.




[28] Some of my fellow reformers objected that, even though the total number of county splits was the minimal number 17, this map split Berks county more than other counties with similar populations. An alternative map that I then drew had only 7.3 D seats for 50% D vote.

[29] The Lieutenant Governor's map was actually based on one of many drawn by the non-partisan computer program of J. Chen, the one that appears first in his expert witness report for the petitioners.

## 4. Can a Fair and Responsive PA Congressional Map Be Drawn for PA?

It is often assumed that both parties can gerrymander to create a map biased in their favor. In Pennsylvania, this assumption was supported by the claim that Democrats had drawn a map that would give them 13 congressional seats and the Republicans only 5 (Leach, 2014). However, that would have required nearly 55% Democratic statewide vote. At 50% statewide vote, that map gives 8.3 Democratic seats and it requires 50.8% D vote for half the seats, still biased in favor of Republicans. The previous section found that the neutral, traditional criteria give about 7 Democratic seats. The intentionally gerrymandered 2011 plan gave 5 Democratic seats for three consecutive elections, suggesting that the most that Republican gerrymandering could do by violating the traditional criteria was to swing the balance by two seats in their favor. It should therefore not be surprising that heavy Democratic gerrymandering could only swing the balance by two seats in the other direction. However, that would result in 9 Democratic seats, which would actually seem to be fair.

Figure 4 shows a map, which is designated N9, drawn by the author with two goals in mind. The primary goal was to draw as many responsive districts as possible .[30] The secondary goal was to maximize Democratic seats. Table 1 lists salient preferences, 9.1 D seats at 50% D vote, 49.9% D vote for half the seats, very slightly biased in favor of Democrats, and it has 8.6 responsive

[30] Interestingly, it has been reported that it is mathematically possible to make all districts equally competitive (Soberon, 2017), although that paper mistakenly describes that result as unfair when it is actually fair and completely responsive winner-take-all.

districts and an R value of 5, significantly larger than historical values (Goedert, 2014). Table 1 also compares these preferences to those of other maps, including the one discussed in the preceding paragraph, which will be designated the Democratic Best Gerrymander (DBG). N9 is better for Democratic voters than DBG, at least for the 7s data base, and N9 is about equally responsive as DBG.

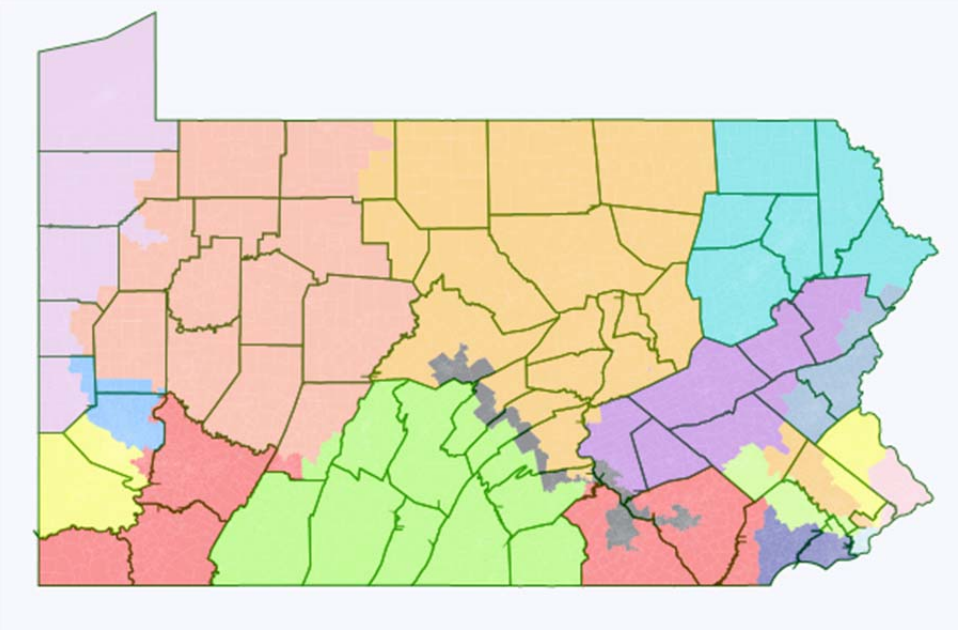


Figure 4. A fair and responsive map (N9) for congressional districts in PA.

The most striking feature of map N9 in Fig. 4 is the long, skinny district 11 with one end in the center of the state; it includes four small regional cities.[31] This district is very slightly Democratic with a Democratic seat probability of 0.5 with 49.3% D statewide vote.[32] To satisfy contiguity, several

[31] One might argue that this district puts together communities of similar interest, namely, city voters who would otherwise be in the minority compared to rural voters surrounding them.

[32] This district is in a part of the state where it has been thought that one could not find a competitive district, much less a Democratic leaning one.

less populous counties are split three ways and it generally violates the traditional criteria as badly as the infamous “Goofy-kicking-Donald Duck” district 7 in the Republican gerrymandered 2011 map. A different map (N3) morphs district 11 into a more compact district that does not split so many counties by not including the furthest fourth city; N3 remains responsive, but leans Republican with 50.3% statewide D vote for 0.5 seat probability.

District 11 in map N9 in Fig. 4 shows the extent to which one might have to go to provide a map that is very nearly fair and quite responsive by drawing in pools of more progressive voters. In contrast, the major reductions in bias and gains in responsiveness are achieved by unpacking city voters. The largest difference is in the Southeastern part of the state which includes the county of Philadelphia and three neighboring counties, as well as parts of two next neighboring counties that comprise a region that has six congressional districts. This region elected three Democrats in three heavily packed districts under the unconstitutional 2011 plan. It has a D preference of 3.95 seats under the SCOPA plan. The D preference in my N3 and N9 plans is 4.99 seats. This gain is due to splitting Philadelphia more than the traditional criteria allow. Only one district, that satisfies the VRA, is totally contained within Philadelphia. Four other parts of Philadelphia are combined with parts of the surrounding counties to form four responsive districts with three leaning Democratic. These districts are somewhat elongated, although they are more regularly shaped than

the “Goofy-kicking-Donald Duck” district 7 in the Republican gerrymandered 2011 map.[33]

Based on my work in this section, I suggest that one can not draw a PA congressional map that significantly favors Democrats without even more grossly distorted districts than in my N9 and N3 maps.[34] This suggests that the practical range of S/V seats bias for PA congressional districts is from 5 D seats to 9 D seats with more than 7.5 and less than 6.5 requiring deviations from the traditional criteria of compactness and not splitting political boundaries. This section demonstrates, however, that it is possible to draw a fair and responsive map for the PA congressional districts if one is willing to bend the traditional criteria.

[33] These districts combine suburban and exurban voters and city voters in nearly equal proportions. Such groupings might reduce political polarization and the influence of narrow interests.

[34] McGann et al. (2016, p.103) draw a similar conclusion about Illinois. However, it is important to acknowledge that the only way to prove such an assertion is to draw all possible maps, but that is essentially impossible given the astronomical number of them (Chikina et al. 2017). On the other hand, it is possible to disprove this assertion by someone or some computer drawing a counter-example map and map drawers should be encouraged to try.

| Map | Data | S | V | R | $R_D$ |
|---|---|---|---|---|---|
| SCOPA | 2016P | 7.7 | 52.7 | 2.4 | 4.4 |
| SCOPA | 2016S | 7.3 | 52.9 | 3.2 | 5.8 |
| SCOPA | 2016Row | 7.4 | 52.7 | 3.3 | 5.9 |
| SCOPA | PVI | 6.7 | 54.1 | 2.7 | 5.0 |
| SCOPA | 7s | 7 | 53.1 | 3.4 | 5.6 |
| D house | 7s | 7.2 | 53.0 | 3.1 | 5.2 |
| D Senate | 7s | 7 | 53.6 | 3.0 | 4.9 |
| D Lt. Governor | 7s | 6.8 | 52.9 | 4.0 | 6.7 |
| R leaders | 7s | 6 | 54.9 | 2.5 | 4.0 |
| Petitioners A | 7s | 7 | 53.2 | 3.5 | 6.0 |
| Petitioners B | 7s | 7.1 | 53.2 | 3.2 | 5.5 |
| Holt | 7s | 6.6 | 53.4 | 3.5 | 5.8 |
| Author N8 | 7s | 7.6 | 51.9 | 4.1 | 7.0 |
| Author N9 | 7s | 9.1 | 49.9 | 5.0 | 8.6 |
| Author N3 | 7s | 8.6 | 50.3 | 4.9 | 8.3 |
| BDG | 7s | 8.2 | 50.8 | 5.0 | 8.7 |

Table 1. Districting plans are listed in the first column and the second column lists the past election data base used. For bias, the S column lists the number of Democratic seats at V=50% (total number of seats is 18), the V column lists the Democratic vote percentage for half the seats. For responsiveness, the R column lists the responsiveness from the S/V graph at V=50%, and the $R_D$ column lists the number of responsive districts obtained by summing the responsiveness function in Fig. 1 over all districts at 50% statewide vote.

## 5. How and why reformers partly miss the mark

There is a vigorous redistricting reform movement in Pennsylvania.[35] It has commendably brought the issue to the attention of the public with many presentations and letters to editors. Together with reform-minded legislators, an amendment to the PA constitution (SB22) was introduced for both congressional and legislative redistricting reform. Concurrently, the LWV brought the successful lawsuit against the PA congressional plan of 2011 which brought much attention to the issue and resulted in the SCOPA map.

The primary reform in SB22 was to create an independent redistricting commission following the California model to replace the legislative commission prescribed in Article II, section 17 of the PA constitution. The traditional criteria of compactness and not splitting political boundaries were already in section 16 of the PA constitution and were strongly supported by the reform movement.[36]

The obvious question, in view of the analysis in this paper, is why should PA reformers wish to prevent an independent commission from trying to achieve fairer and more responsive redistricting plans by constraining it to adhere to the traditional criteria? This is hardly a new question. Based on their simulations (Chen and Rodden, 2013) wrote “Rather, we

[35] The face of this movement has been Fair Districts PA (FDPA) which has included League of Women Voters (LWVPA) and Common Cause (CCPA). The author has been a member of a CCPA redistricting team that has debated the issues in this section.

[36] Actually, there was an awkward drafting oversight as these traditional criteria only apply to legislative redistricting in Section 16 of the PA constitution, which does not mention congressional redistricting.

suggest that unless they are prepared to take more radical steps that would require a party's seat share to approximate its vote share, reformers in many states may not get the results they are expecting."[37] The empirical analysis in this paper strongly supports these authors' simulations and thereby reiterates the question.

An answer is that many reformers consider it repugnant even to allow consideration of partisanship in reform.[38] In addition, the PA bills to reform the PA constitution, SB22 and HB722, were actually much worse than simply ignoring partisan bias and competitiveness. Those bills would have prohibited their independent commission from even considering partisan fairness and competitiveness. They did this by prohibiting the commission from even looking at past election data and party registration.[39] The commission would thereby have been prohibited, not just by statute, but by the PA constitution, from estimating the bias of its map.[40] These prohibitions would effectively have precluded future court challenges like the successful LWV suit whose expert witnesses extensively used such data. How could a court consider such a suit that used election

[37] Chen and Rodden in a NY Times piece 1/24/2014 based on computer simulations adhering to the traditional criteria in (Chen & Rodden, 2013). In strong support of this statement, Chen and Cottrell (2016) reported that unintentional gerrymandering accounted for all but half a seat in the PA 2011 plan. In contrast, as an expert witness in the LWV case, Chen's use of the median minus mean measure indicated that intentional gerrymandering accounted for about 2/3 of the bias in the PA 2011 plan. My analysis of the data in that 2016 paper agrees better with this latter result, giving half the bias of the 2011 plan.

[38] This ignores the reality that elections are inherently political activities that, for better or worse, are strongly partisan.

[39] SB22 section (j)(3)(ii-iii).

[40] It is a bit like manufacturing a product and not testing it for foreseeable consequences before selling it.

results to show unfairness if the map had to be drawn without considering such data? Although many reformers think the use of those data should be abjured because they have been used to gerrymander for unfair partisan advantage, that is not a good reason to prohibit their use by an honest independent commission which could use such data to "anti-gerrymander"[41] to bring about greater fairness.[42]

Nevertheless, many reformers prefer a non-partisan process, but they mistakenly proclaim that traditional map drawing criteria necessarily entails a fair process, ignoring its being the "myth of non-partisan cartography".[43] This paper adds to the political science literature by showing that adherence to the traditional criteria will not provide a fair process for the congressional districts of Pennsylvania. Good public policy is generally based on outcomes determined by the best social and scientific analysis. It would seem that continuing to advocate for non-partisan cartography is to deliberately advocate for poor public policy.

Of course, there are other important aspects to redistricting reform. A commendable one is to prevent political leaders from punishing non-conforming incumbents by drawing them out of their

[41] This is a possible term to succinctly describe what this paper advocates. As gerrymandering draws lines to obtain partisan advantage, anti-gerrymandering draws lines *not* to obtain partisan advantage. Unintentional gerrymandering draws lines with no intent to address partisan advantage, but with the result of partisan advantage.

[42] And its availability publicly would allow unscrupulous members of the commission and the consultants it hires to surreptitiously use it while keeping honest members and the public in the dark.

[43] Taylor and Gudgin (1976) remarked "Perhaps the main conclusion of our analyses is that the myth of non-partisan cartography hides the real issues of democratic representation."

districts. Another consideration is how to accommodate communities of interest.[44] These considerations would presumably be dealt with honestly by an independent commission which would balance them with fairness and responsiveness.[45] In the opinion of this author, what matters most is fairness and responsiveness to voters. The import of this paper is that these will not be achieved in Pennsylvania without actively trying to overcome its political geography.[46]

## 6. Looking forward.

There is considerable current interest in writing new laws for redistricting in Pennsylvania and elsewhere. Given the present understanding of the public and the reluctance of a dominant party to relinquish any advantage it has due to political geography, it is likely that new legislation will embody the traditional criteria rather than the bolder criteria of fairness and responsiveness. While this will produce fairer and more responsive outcomes than the most heavily gerrymandered

[44] Of course, communities of narrow interest, like those that try to keep open obsolete defense installations, may not be in the greater public interest. Also, some supposed communities of interest, like large cities, might be better served by being split so they would have more representatives to appeal to for worthy needs (Hirsch and Ortiz, 2005), and there would be a greater chance for one of those representatives to be on a relevant congressional committee.

[45] Cain (2012) has reviewed independent commissions and the balancing of criteria issue. Balancing many criteria, not just the traditional criteria, fairness and responsiveness, was exhaustively discussed by Butler and Cain (1992).

[46] Implementation for achieving fairness and responsiveness should involve evaluating many maps for these properties, including those drawn by computers and by citizens, as well as those drawn by a commission.

plans, a concern is that the public will be disappointed with outcomes that still remain unfair, and many citizens will then question whether all attempts for redistricting reform are futile. One purpose of this study is to help educate people as to the likely outcome of traditional reform.[47] Even if traditional reform is all that can be achieved at the present time, at least people will know why it did not live up to expectations and not conclude that redistricting reform is necessarily futile.

Even if it is unlikely that fairness and responsiveness will completely replace the traditional criteria in the near term, it may be worth thinking about how some progress in that direction might be brought about in new redistricting legislation. I suggest that a reform law begin with a preamble "section P" that would include language like "(a) Representative government depends on elections that are fair and responsive to voters and (b) For elections to serve their function, congressional districts must be drawn consistent with (a)." Note that (a) emphasizes voters, not parties, and would seem to be unobjectionable. A later detailed section on criteria would list the legal criteria, like equal population, ending with a clause about the traditional criteria "District boundaries shall be compact and coincide with the boundaries of political subdivisions of this state to the extent that this is consistent with the general policy in Section P." The crucial qualifier "to the extent …" means that political geography would not be allowed to silence commissioners more concerned with fairness and responsiveness, and it would also allow citizens to submit

[47] Another purpose was to convince myself of what many political scientists have been saying for years, and to do so in the particular context of Pennsylvania congressional districting.

maps that have a few more splits of political subdivisions than the absolute minimum. Assuming that the process would be transparent and that past election data could be used, the public could then evaluate the commission's draft map and call upon it to choose a fairer and more responsive one if need be. Whether this would be effective would depend upon the vigor of advocates of fairness and responsiveness compared to that of advocates for other criteria.[48]

Acknowledgements: The author especially thanks Dave Bradlee for providing and maintaining a convenient website for drawing and evaluating maps and for assisting me to add my 7s data base to his app. Thanks are due to Amanda Holt for providing some of her maps in this format, Michael Waxenberg who put some of the *amici* maps in this format, Michael Johnson for help acquiring the 2016 election data and Lora Schoenberg for help with the 7s data base.

[48] Stephanopoulos (2013) mentioned this in a study that showed the relative ineffectiveness of such efforts historically. It may also be noted that many of his examples already had low bias, so significant improvement would not have been expected. Similarly, some states may not have the political geography that leads to significant unintentional gerrymandering, in which case the traditional criteria should suffice.

## Appendix A. Two other methods for measuring bias

The S/V approach to measuring bias requires some modest math and it involves counterfactual analysis. Courts, especially Justice Kennedy, have asked for a simple standard as a prerequisite for affirmative gerrymandering rulings. This has led to several proposals, two of which are critically discussed in this appendix in the context of their performance on the SCOPA map for Pennsylvania congressional districts.

The efficiency gap (EG) is being advocated as a tool for evaluating fairness based upon only one election result without the need for counterfactuals and construction of the S/V graph (McGhee, 2014). It does, however, propose a normative S/V graph for zero bias. An even newer measure of bias (designated $\delta$) also can be used with just one election result (Warrington, 2017). Interestingly, it does not provide a normative S/V graph for zero bias because it does not simply depend on S and V, but also on how the votes are distributed among districts.[49]

A measure of the intrinsic bias of a map should conform to the principle that it remain constant upon shifts in the statewide vote, at least for likely ranges of the vote. The EG suffers from discontinuous jumps, as has been noted by many critics, and so does the $\delta$ measure. However, these jumps can be eliminated for the EG by assigning statistical probabilities to seats. Nevertheless, both the EG and the $\delta$ bias values vary considerably as statewide vote is varied for the SCOPA map, implying that they generally fall short of measuring the intrinsic bias of

[49] This feature is also shared by voter-centric variants of the efficiency gap (Nagle, 2017; Cover, 2018; Tapp, 2018).

maps. It is shown in Appendix B that a measure based on the S/V graph satisfies this test.

### 1. The efficiency gap

The basic efficiency gap, to be abbreviated EG1, calculates the difference in wasted votes between two parties.[50] The original paper (McGhee, 2014) then derived a simple formula in terms of seats and votes

$$EG2 = \Delta S - 2\Delta V \ , \qquad (1)$$

where $\Delta S$ is the fraction of seats won by party A minus ½ and $\Delta V$ is the fraction of the two party vote for party A minus ½. When the number of voters is the same in all districts, EG1 and EG2 have the same value. When this is not the case, namely, when there is turnout bias, it has subsequently been shown (Cover, 2018, McGhee, 2017) that EG2 is the more fundamental measure of bias and this Appendix will use it exclusively. An important aspect of EG2 is that it provides a normative S/V curve which has responsiveness R=2, twice as large as proportionality, and that makes EG2 conform better than proportionality to empirical election data.[51]

One concern with either EG is that its value changes abruptly when a district changes parties with just a small change in the vote for that district, which may even be balanced by an opposite change in the vote of a different district that does

[50] An exhaustive set of variations of the EG has been considered in this journal. (Nagle, 2017)

[51] However, as has been emphasized by the author previously in this journal, fundamental principles lead to proportionality as the ideal S/V graph, so the actual S/V results indicate the irreconcilability of single member district systems with fundamental principles (Nagle, 2017).

not change parties. However, it has been noted that each such abrupt change will be relatively small when there are many districts (Nagle, 2017). Now, it may also be noted that abrupt changes in EG2 can be eliminated entirely if one uses the probabilistic estimate of a district's seat assignment in Fig. 1. This essentially uses the S/V curves in Fig. 2 to calculate EG2. Although this runs counter to the desire to eliminate counterfactuals from measures of bias, it overcomes the many critics who have condemned the EG on this ground. Nevertheless, there is a more substantial criticism of the EG to which we now turn.

Figure 5 shows the values of EG2 versus statewide vote for the SCOPA map using five data bases as elections. The concern here is that the values of the EG vary systematically with V, indicating less bias when the political winds favor Democrats.[52] It is then problematic to assign intrinsic bias to the map using the EG. The smaller EG for larger V in Fig. 5 is simply related to the SCOPA plan having greater responsiveness R (approximately 3 in Table 1) than the normative EG value of 2. As there is nothing wrong with a map having high responsiveness, this is the reason that one should not apply the EG, which includes a normative value of responsiveness, when evaluating the bias of particular maps for redistricting.[53]

[52] Best et al. (2018) have previously found a similar result for North Carolina and emphasized this concern.

[53] However, in its favor, consider the solid squares in Fig. 5 which show EG values for the actual statewide vote from each past statewide election. All five EG values indicate that the SCOPA map is biased in favor of Republicans. This is a stronger statement than what could be immediately drawn from the same data in Fig. 2. It might also be mentioned that using strict proportionality with R=1 leads to even more variation in its estimate of bias in the SCOPA map than the EG.

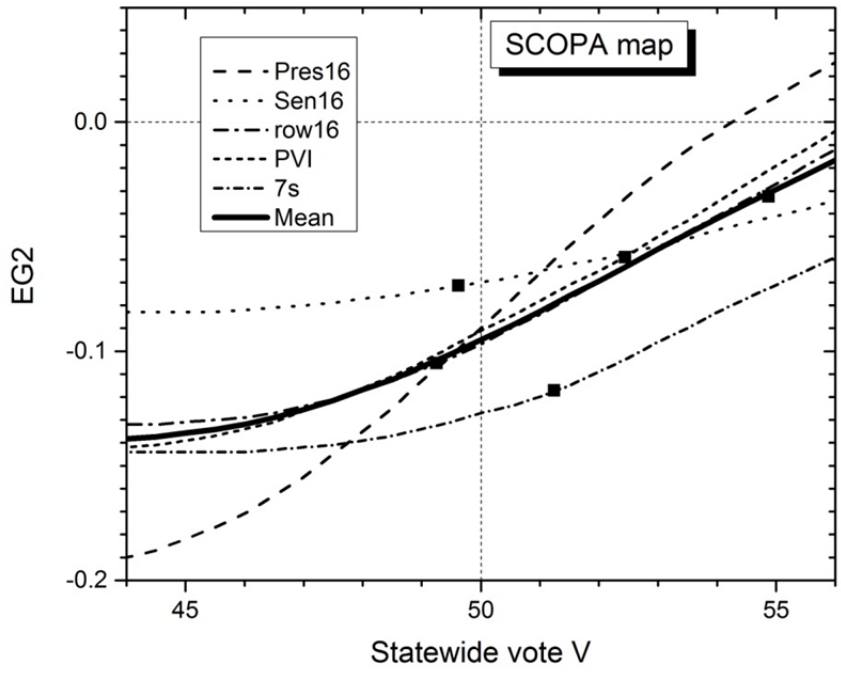



Figure 5. Efficiency gap EG versus statewide vote V for five elections shown in the legend and for the mean EG from these five data bases. The squares show the statewide vote of each of the five data bases. Negative values of the EG indicate bias in favor of Republicans.

## 2 Declination measure of bias

We turn next to a measure of bias, recently proposed in this journal, that is also worthy of consideration (Warrington, 2017). Fig. 6 shows the definition of the declination measure, denoted $\delta$, which is proportional to the difference in the declination angles for the districts won by the opposing parties. My experience with such figures is that $\delta$ does indeed

correlate with being positive when there is bias against party P.[54]

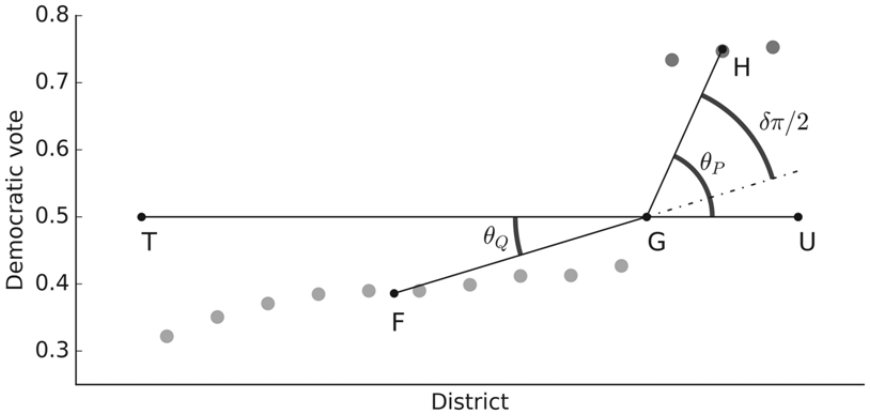



Figure 6. Districts are ranked by least Democratic vote on the horizontal axis with the vote given on the vertical axis. The coordinates of point F (H) is the average district rank and average district vote won by Republicans (Democrats, respectively) and point G is placed midway between the ranks of the two groups of districts at V = 0.5. (Fig. 1 in (Warrington, 2017))

Fig. 7 shows δ bias for the SCOPA map using the two 2016 data bases that differed most in the statewide vote. It is disconcerting that δ indicates opposite bias for the two data bases, strongly favoring Republicans for the 2016 Senate data base and slightly favoring Democrats for the 2016 row offices. If this difference were due to different geographical distributions of voters in the two data bases, the results would

[54] Interestingly, though, it was been stressed that an advantage of the δ measure is that it does not depend uniquely on S and V. However, this means that δ violates the principle (McGhee, 2017) that a valid measure should register more bias when a party receives more seats with the same vote because, as Tapp (2018) has recently proven mathematically, any measure that, like the δ measure, depends on variables other than seats and votes, violates McGhee's principle.

be much different when the vote is shifted to the same value, but Fig. 7 shows that the curves for the two data bases track each other rather well.[55] The rapid decrease in δ as the statewide vote increases in the range of votes that is most relevant to PA does not bode well for this as a measure of a plan's bias.

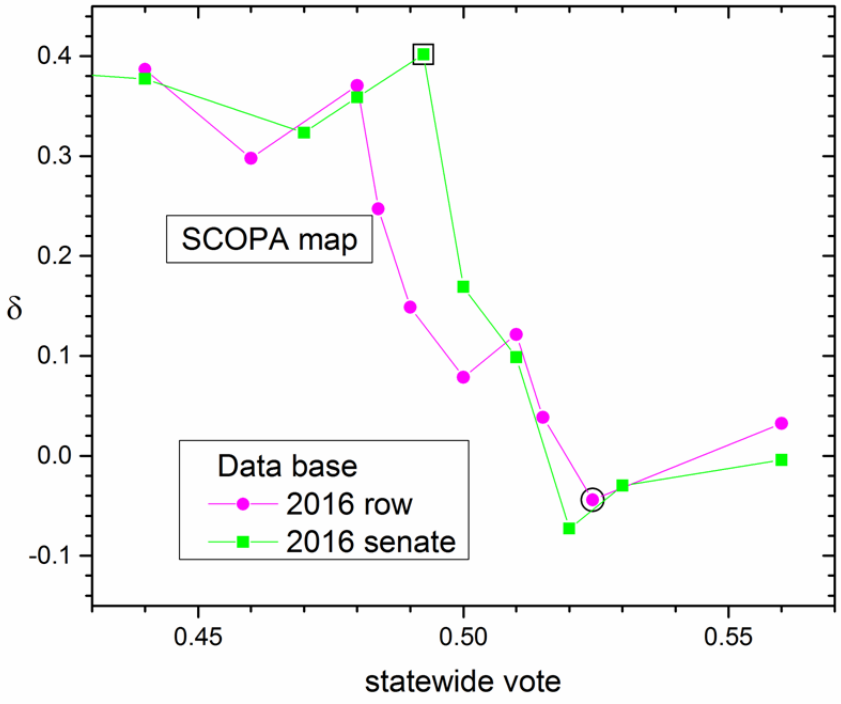



Figure 7. The value of declination bias δ for the SCOPA map using the data bases in the legend. Positive values indicate bias in favor of Republicans. The points for the actual statewide votes are encompassed by large open symbols. Other values were obtained by uniform shift of district votes.

This paper's fundamental criticism of both EG and δ, from the point of view of measuring the bias of a map rather than measuring the bias of a specific election, is that their biases vary systematically with the vote V. Within the realistic range

[55] The graphs of δ exhibit small abrupt changes as a seat swings to another party. It is unclear how to eliminate these, in contrast to how that can easily be done for the EG.

45-55% of PA statewide D vote, Fig. 5 shows that the EG in favor of Republicans is greater for smaller D vote. The $\delta$ measure in Fig. 7 even changes sign when the vote increases to D vote of 52.44%.[56]

## Appendix B. Using the S/V curve to measure bias for any statewide vote

Appendix A ended with a criticism that the EG and $\delta$ measures did not do well on what will henceforth be called the variable votes test, namely that robust measures of a map should not be too sensitive to the statewide vote.[57] It would be hypocritical not to apply this test to the S/V method preferred in this paper. However, the variable votes test is impossible for the S/V seats measure in Section 2 because that is applicable only for one value of the vote, 50%. Similarly, the S/V votes measure uses only that one vote that gives half the seats. Furthermore, the seats measure applied at 50% statewide vote is especially vulnerable to criticism in states where the typical statewide vote is far from 50%. For those states, it is more appropriate to apply a measure in the range of the expected statewide vote. This Appendix will first show a method for how this can be done using the S/V curves. The variable votes test will then be applied to this S/V curve method for the SCOPA map for Pennsylvania.

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[56] In order to obtain a definite value for either the EG or $\delta$, one might consider averaging over all the existing data sets or, similarly, taking their average at the average statewide vote.

[57] Cover (2018) has called this the sensitivity test. Cover (2018) and Best et al. (2018) have argued that the EG does not generally satisfy it because the EG conflates bias and responsiveness.

Figure 8 shows two S/V curves. One is for the Democrats and one is for the Republicans. The two curves come from the same election data base and are therefore simply related. The number of Republican seats is just the total number 18 of seats minus the number of Democratic seats and the Republican statewide vote % is just 100 minus the Democratic vote %. This means that each point on one curve is as far from the center of the figure, which is located at 50% vote and 9 seats, as is a point on the other curve in the opposite direction from the center; in mathematical terminology, the two curves are related by a geometric inversion. When the two curves are identical, the common curve has inversion symmetry; it is unbiased by the symmetry standard because the plan treats both parties the same.[58]

Differences between the two curves in Fig. 8 can be used to define measures of bias.[59] Here, we use a simple seats based definition; for vote V, it is the difference in number of seats between the two curves, divided by twice the number of districts to give the usual seats based value at V = 50%. This extended S/V measure will be designated $B_{GS}$ for Bias of Geometric Seats.[60]

[58] (Grofman and King, 2007).

[59] One such measure is mentioned in footnote 21. Instead, the measure adopted here is quite similar to the "specific asymmetry" recently introduced by Baas & McAuliffe (2018), which also uses S/V curves and their inversions (called reflections in that MS). One difference in implementation is that their S/V curves were not obtained using statewide past election results.

[60] It may also be noted that one could define an extended votes focused measure of bias $B_{GV}$ by taking half the difference in votes between the two curves at any fixed number of seats; this makes it the same at V=50% as the S/V vote bias in Section 2.

Figure 8. Seats/Votes curves for both parties for the SCOPA map using the 7s past election result data base. The curves are related by inversion about the center at V=50% and S=9. Also shown is the $B_{GS}$ bias.

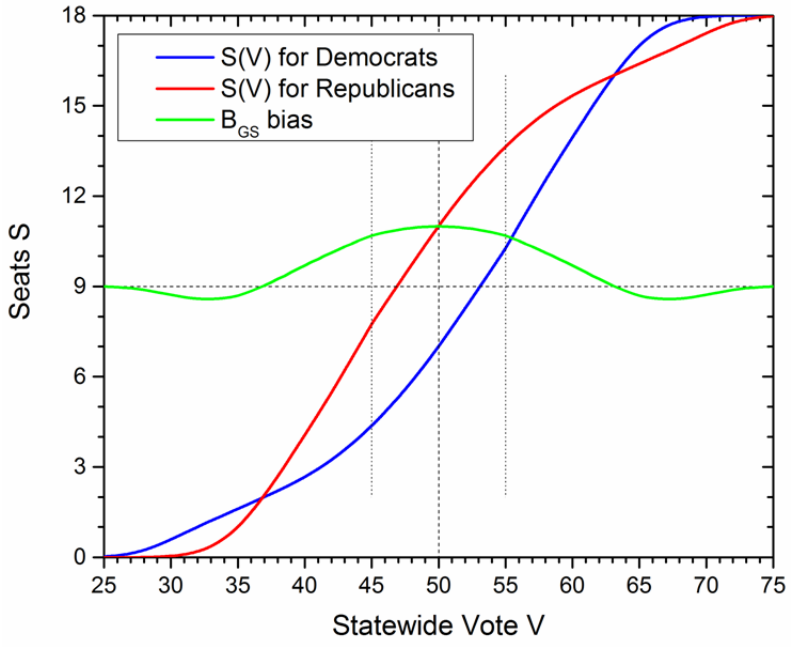
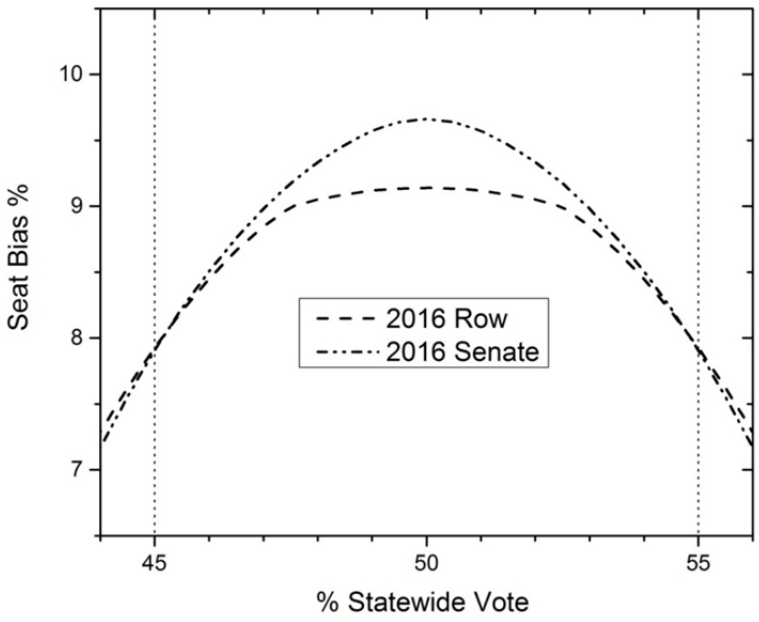



Figure 9 shows the values of $B_{GS}$ for a range of vote V for the SCOPA map for the same two data sets used in Fig. 7. These values are relatively insensitive to V in its most probable PA range of 45% to 55%, varying by only about 15% in this range. This implies that $B_{GS}$ is a much more robust measure of bias than the EG, which varies by more than a factor of 2 in Fig. 5, or the $\delta$ measure which even changes sign in Fig. 7.

Figure 9. Seat bias $B_{GS}$ obtained from S/V curves *vs.* statewide vote using the 2016 row offices and the 2016 Senate past election results data base. The vertical dotted lines indicate the relevant range of statewide vote in PA.



## Appendix C. Was the 2016 Presidential Election in PA Anomalous?

The 2016 presidential election predicts more D seats at 50% of the vote than the other data bases. It has been opined in the press that this might have been because the SCOPA map was purposely drawn using this particular data base. In any case it is interesting just how this data base differs from the other 2016 data bases. For each of the two data bases Figure 10 shows the statewide vote for each district at which its partisan preferences are equal. A district is then most likely to flip from Republican to Democratic when the statewide vote increases from smaller to larger than that district's preference.[61] Graphs like the two in Fig. 3 will henceforth be called flip graphs.

Figure 10 compares the flip graph for the 2016 presidential data base to the one for the 2016 row offices data base. The differences in the flip vote for safe Democratic districts 2, 3, and 18 and for safe Republican districts 13, 12, 15, 11, 9 and 16 are of little consequence. Of the remaining districts 1, 4, 5, and 6 have flip votes smaller for 2016P than for 2016Row because they voted relatively more for Clinton than for the other D candidates. These districts are located around Philadelphia in the Southeastern part of the state. District 8 in the coal mining Northeast and districts 14 and 17 in the coal mining Southwest voted relatively more for Trump. There was little difference for district 10 which includes the state capital and district 7 which has several metropolitan areas. The graphs for the other data bases, which are not shown for clarity,

[61] In (Nagle, 2015) graphs like Fig. 10 were called seats/votes graphs, but that didn't take into account uncertainty in the single districts that are accounted for using Fig. 1. The preferences in Fig. 10 have been shifted to correspond to 50% statewide vote.

are similar to the 2016 row office data base. This suggests that the 2016 presidential data base is an outlier due to the particularities of that election and therefore does not reflect the true preferences of PA voters. If so, then the SCOPA map may prove to be a disappointment to Democrats.[62]

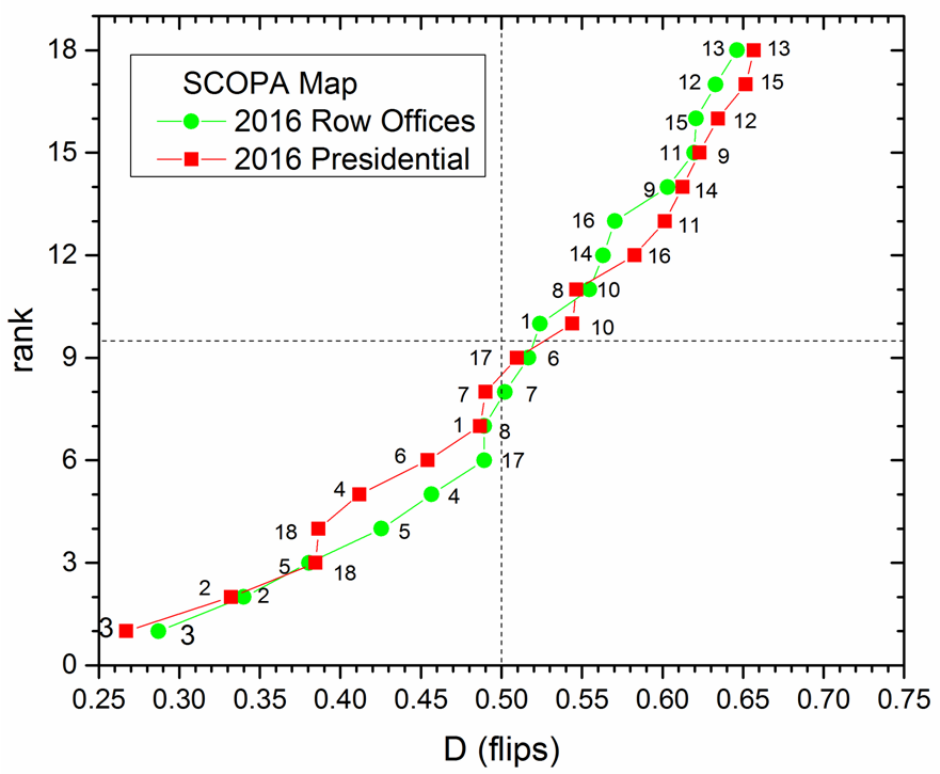



Figure 10. The districts are rank ordered along the vertical axis by increasing Republican preference. The numbers of the districts on the SCOPA map are shown next to each data point. The horizontal axis shows the statewide Democratic vote at which a district is most likely to flip from Republican to Democratic using the proportional shift method. The data bases are shown in the legend.

[62] Some media reports when the SCOPA map was unveiled suggested that it is biased in favor of Democrats, but that is clearly inaccurate even using the 2016P data base. One report estimated that it would give over 8 Democratic seats, but it was unclear what fraction of the D vote would be required for that estimate.

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(Chikina et al., 2017)(Cover, 2018)(Gelman and King, 1990)
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(Soberon, 2017)(Stephanopoulos, 2013)
(Stephanopoulos and McGhee, 2015)(Chikina et al., 2017)
(Tapp, 2018)(Taylor and Gudgin, 1976)(Warrington, 2017)