Comparing Metrics for Partisan Bias in the US Congressional System: Proportionality Versus Traditional Seats Bias

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Abstract: Proportionality has been considered to be a desirable norm for representation even though it is generally unachievable over significant ranges of the vote in the single member district system. Nevertheless, it is appealing to measure bias as a deviation from proportionality, but just at the projected average partisan vote. However, this paper argues that this modified proportionality is not a good metric of bias based on analysis of all 2022 US Congressional plans. It is inconsistent as the overall two-party vote swings. It most often exaggerates the bias in unbalanced states. It leads to anti-majoritarian results. It ignores that the single member district system promotes greater responsiveness with a larger fraction of competitive districts, which is considered by many to be a desirable feature. It unduly shames states with a strongly dominant party when the political geography of the state prevents the achievement of proportionality.

In contrast, this paper finds that the traditional seats bias metric, which is based on the principle of equal seat share for equal vote share, is a robust and adequate metric of partisan bias for most states, even for unbalanced states. The seats bias uncertainty is typically less than 1%, and the difference in partisan bias between two maps is even better determined. As is well known, responsiveness is also an important consideration, and several forms of it are used to sort the 2022 districting plans into three broad types.

Introduction

New redistricting legislation ought to consider partisan fairness since legislative bodies, especially Congress, have become so rigorously partisan. SCOTUS has abdicated oversight of partisan bias to the states[1], so any action will have to take place in state election law (Cervas et al., 2022). In the event that some states might consider changing their election law for redistricting for the next cycle, it is important that the issues be thoroughly discussed in a timely fashion, especially because the lead time can be quite long in states that would require changing their constitutions. For example, a bill to change the PA constitution has already been introduced in the state house.[2] This bill specifically includes partisan fairness as a criterion to be considered.

This paper addresses the particular issue of how to measure total partisan bias in districting plans.[3] Although it is clear that partisan bias should be minimized, which of the many metrics should be used has been rather unclear and even contentious. This has weakened the case for including partisan bias in election law, instead falling back on the so-called neutral criteria, such as compactness and not splitting counties. While the neutral criteria prohibit the worst gerrymanders, in many states they also make it nearly impossible to obtain partisan fairness (Keena et al., 2021, Rodden, 2019, Nagle, 2019). An interesting new criterion even works against partisan fairness in half the states (DeFord et al., 2022). Even when the neutral criteria permit a fair map, they allow maps that cover a fairly wide range of partisan bias.[4] In that case, the map that minimizes partisan bias could be chosen, while still satisfying other criteria, but for that it is still necessary to have a good metric.

It has often been supposed that, when the percentage V_A of voters vote for party A, then party A should normatively obtain the same percentage of the seats $S_A = V_A$. This way, even though

[1] Rucho v. Common Cause, 139 S. Ct. 2484 (2019).

[2] Pennsylvania House Bill 1776 Session 2023-2024 introduced April 17, 2024. § 19. (2) A redistricting plan may not: (ii) Provide a disproportionate advantage to any political party.

[3] It is important to note that our goal is to measure the total bias, whatever its provenance, and not gerrymandering, which may be defined as the bias that ensues from deliberate attempts to achieve partisan advantage above whatever "natural" bias can be attributed to the other causes (Barton & Equia, 2024), such as the political geography of a state or its districting rules.

[4] Goedert et al. (2024) have recently employed "hill-climbing" simulations that exhibit the "stealth gerrymandering" effect with respect to compactness.

some A voters will live in districts represented by other parties, A voters as a group have the same average representational power, defined as S_A/V_A, as voters for a different party that obtains the same percentage of seats as its overall vote percentage. This yields proportionality which is represented by the seats-votes function $S_A = V_A$. It has been achieved by being baked into various voting systems.[5] The proportionality metric of partisan bias, P_A, is then given as deviations from proportionality, $P_A = S_A – V_A$.

The previous paragraph assumes that the power of party A is proportional to its percentage of seats S_A. However, this is a debatable assumption because a party's power clearly decreases abruptly when it loses a majority of the seats. Interestingly, if one assumes that party power is proportional to S_A^{ω}, where the exponent ω may be different than 1, then the requirement that voters in both parties have the same average power S_A^{ω}/V_A for party A and $(1- S_A^{\omega})/(1- V_A)$ for party B results in a bilogit seats-votes curve (King and Browning, 1987) that is quite different from proportionality when the ω exponent is not equal to one. One example is that choosing ω = 1/3 results in the classic cubic "law" (Kendall and Stuart, 1950). Party power according to this choice of ω is at least as realistic as the assumption $\omega = 1$. Therefore, contrary to previous assertion (Nagle, 2017), the previous paragraph does not provide a privileged normative foundation for proportionality.

In any case, proportionality is clearly not achievable in the single member district voting system in the United States (Grofman, 1982). Parties with 1% of the vote in a two party state cannot possibly have any district with more than 50% of its supporters when there are fewer than 50 districts, even if all those voters comprise 100% of the population in one geographical area. More realistic political geography makes it highly likely that a political minority will not obtain representation in a two party state even with 30% of the vote and it would take an exceedingly generous majority party to accept a map that would likely obtain such representation. An important paper definitively and rigorously showed that it was mathematically impossible to draw a single district that would lean Republican in the decade before the MA map was drawn even though the vote was nearly 40% Republican (Duchin et al., 2019). That is because the partisan voter preference for MA is geographically relatively homogeneous outside the Boston

[5] The list systems and the mixed member proportional system used in other countries are notable examples (Farrell, 2011).

area. Likewise, it is important to consider a hypothetical state that has completely homogeneous partisan preference geographically for two major parties. For such a state, all districts, no matter how drawn, would have the same partisan preference and the seats-votes S(V) curve would necessarily have zero seats for a party when it has V less than 50% and all the seats when V is greater than 50%.[6] Empirical studies (Goedert, 2014, Goedert et al., 2024, Barton, 2022) have shown that "partisan-neutral maps rarely give seats proportional to vote" (Rodden and Weighill, 2022).

Even though it is not generally possible to achieve proportionality in severely unbalanced and relatively homogeneous states, proportionality has remained appealing, so a fall-back goal is to draw maps that can be estimated to most closely achieve proportionality at a value of the statewide vote estimated from previous elections. Proportionality in this modified form is consistent with Justice O'Connors characterization in *Davis v. Bandemer* (1986)[7] that "the plurality opinion ultimately rests on a political preference for proportionality - not an outright claim that proportional results are required, but a conviction that the greater the departure from proportionality, the more suspect an apportionment plan becomes", and to a 2015 amendment to the Ohio state constitution that the "statewide proportion of districts … shall correspond closely to the statewide preferences of the voters of Ohio."[8] The modified metric of bias mP is then defined by how far the seat share differs from the vote share, mP = S(V) – V where V is the estimated voter preference in a state and S(V) is the estimated seats when the vote is V. It is far less demanding of a map to require that it make mP equal to zero because it only has to do this for one estimated voter preference V rather than make the entire seats/votes curve S(V) equal to V for all possible votes. Proportionality in this form has been clearly described and strongly advocated (Duchin and Schoenbach, 2023). Recent papers in these pages have essentially approved the mP metric (Ramsay, 2023, Gordon and Yntiso, 2024). Furthermore, a popular online redistricting tool, DRA2020 (Bradlee, 2020), prioritizes mP over the many other metrics it calculates.

[6] Of course, even in such a state there would be statistical fluctuations in turnout that would make actual outcomes deviate from the most probable projection represented by this all-or-nothing S(V) function.

[7] Davis v. Bandemer, 478 U.S, 109 (1986).

[8] Article XI, Sections 6(B) of the Ohio Constitution.

Calculations of the modified proportionality (mP) partisan bias of maps are easily obtainable from DRA2020 (Bradlee, 2020). Briefly, DRA uses precinct data from past elections for statewide offices to estimate the Democratic seat share S(V) and 100% - S(V) for the GOP seat share at the actual two-party vote share V.[9] DRA is a useful and easily accessible tool for mP and many other metrics, as well as for calculating entire S(V) curves.

Even though modified proportionality mP would therefore appear to be an attractive and accessible metric for measuring partisan bias, we present arguments that it is a poor one for the single member district system used in the USA. Thus, while partisan fairness should be an important feature in states' redistricting laws, no reference to proportionality should be included,[10] and reformers drawing citizen maps should not try to minimize mP bias when V is not 50%.

In this paper we compare estimates of mP bias to those of the traditional seats bias.[11] We show that seats bias, SB, is a better, and indeed, quite adequate metric of partisan bias, especially for comparing maps. This encouraging finding is likely even more important than our negative conclusion regarding proportionality.

The paper begins by examining the 2022 Virginia Congressional plan. This shows that high responsiveness is at the root of the problem with using proportionality. The next section then shows that high responsiveness broadly pertains to all states. Three undesirable consequences of using the mP metric are then addressed in the next section. A section on seats bias shows that it is a much more robust metric that has small uncertainties. Further comparisons recognize that bias alone should be modulated by responsiveness, and, accordingly, three broad types of state plans are differentiated. General discussion and conclusions follow.

For convenience, the following glossary collects some of the terms employed in this paper.

P is proportionality bias - the difference between % GOP seats S and % GOP vote V.

mP is proportionality P just at the estimated average vote V

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[9] This paper is confined to current USA states which are dominated by two parties, so for convenience we use the DRA two-party vote such that the GOP and DEM vote percentages add to 100%.

[10] For example, the current PA House Bill 1776 prohibits "disproportionate advantage" *vide infra fn. 2,* which could be construed as requiring proportionality.

[11] This metric is also conveniently available from the Advanced section of DRA where it is designated seats bias and mP is designated proportionality at the DRA composite vote.

SB is seats bias defined as GOP seats in excess of 50% for equal vote for the two parties

S(V) is the seats-votes curve defined as seats S for vote V.

$R_{40\text{-}60}$ is a broad responsiveness defined as [S(60%)-S(40%)]/20%.

ρ is responsiveness narrowly defined as the slope in the S(V) curve just at the average V.

ρ_{50} is responsiveness narrowly defined as the slope in the S(V) curve just at V = 50%.

A pertinent and typical example

The 2022 congressional plan for the state of Virginia is considered by applying the results of ten statewide elections to the plan's districts.[12] Each election provides an estimate of voter preferences and the corresponding seat share. Examining the results from such a suite of elections provides a better view of a plan's characteristics than using only one election or a single composite of several elections. Figure 1 shows the values of bias using the proportionality metric.[13] They vary from plus 4% bias for the G21 election that had 49% Democratic vote, where the plus sign favors the GOP, to minus 11% for the S18 election that had 58% Democratic vote, where the minus sign declares that the map is biased favoring Democrats. Such wide and systematic variations imply that proportionality does not measure a valid bias because the bias of a plan is an intrinsic property of the plan that shouldn't vary with swings in actual elections. DRA alleviates this somewhat by taking an average which is shown in Fig. 1 as the DRA composite, but the standard error of the mean shown as the error bar on the open square is quite large, consistent with differences that would occur if different elections were chosen to compile a composite election.[14]

Figure 1 also shows results from another bias metric, the traditional seats bias metric, designated as SB in this paper.[15] In contrast to proportionality, it has more nearly the same values for different two-party votes V_D, so its uncertainty is much smaller than for mP. Of

[12] The VA elections were those available in the DRA database (Bradlee, 2020), president 2016 and 2020 (P16 & P20), senate 2018 and 2020 (S18 & S20), governor, lt. governor, attorney general 2017 and 2021 (G17,G21,LG17,LG21,AG17,AG21).

[13] These data were obtained from the advanced section of the current (2024) version of DRA2020 where what is designated as mP in this paper is titled proportionality in DRA and α_S is titled seats bias.

[14] The DRA composite only uses six of the ten elections in Fig. 1. Other analysts often use only one or two elections, usually the latest presidential elections. For the P20 election mP would report even more bias favoring Democrats in Fig. 1 than the DRA composite.

[15] This metric is also conveniently available from DRA where it is designated seats bias

course, the two metrics agree for the elections that had the vote closest to 50% because mP at 50% vote is identical to seats bias. Bias at 50% two-party vote violates the bedrock principle of equal seats for equal vote. However, the DRA composite mP value in Fig. 1 deviates from that because the elections with larger Democratic vote drag the mP average down.

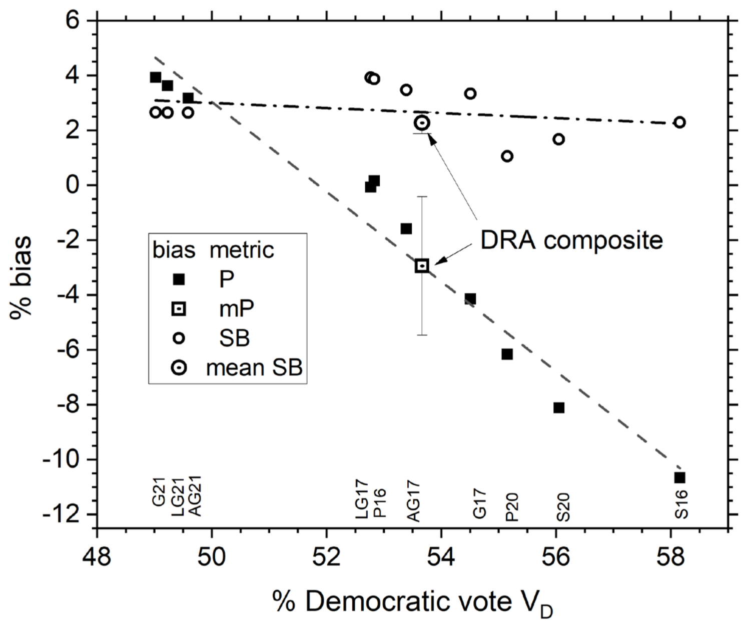


Fig. 1. Proportionality bias P and seats bias SB along the vertical axis for the VA congressional plan of 2022 for the DRA composite election and ten statewide elections identified along the votes axis as in footnote 12.

The reason that proportionality varies so much and so consistently with vote shares is due to the VA plan having a feature that is often considered to be desirable, namely, that it is more responsive to voters than proportionality. Figure 2 is a reminder that ideal proportionality, shown by the bold dashed straight line, would switch 2% of the seats if the overall vote switches by 2%. In contrast, the seats/votes S(V) curve shown for VA in Fig. 2 would switch 4.8% of the seats when the vote switches between 52% and 54%.[16] When the S(V) curve is more responsive than proportionality, then that curve automatically predicts that the proportionality metric becomes more negative as V increases because it equals the difference V – S(V) between proportionality and the S(V) curve.[17]

[16] This is even greater than the responsiveness of the ideal efficiency gap (EG) which idealizes changing 4% of the seats for a 2% vote swing.

[17] Using V – S(V) instead of S(V) – V conforms to the DRA sign convention that the GOP is favored by a positive value and the Democrats are favored by a negative value.

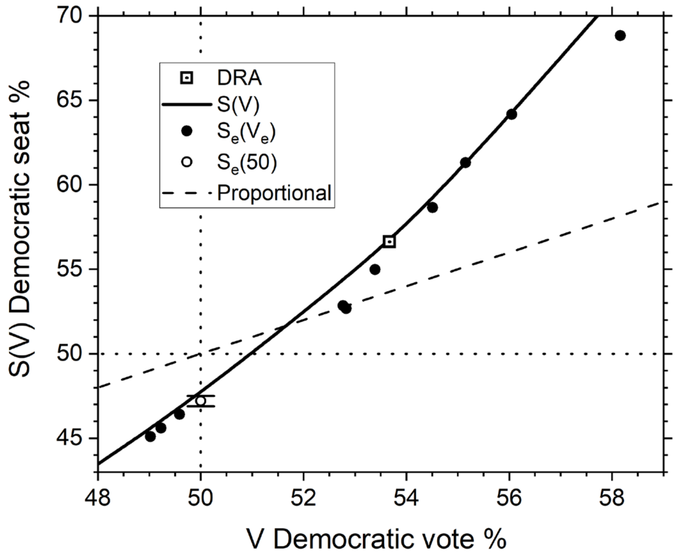


Fig. 2. The seats-votes S(V) curve (solid) for the Virginia congressional plan of 2022 produced from the DRA composite election (open square). The solid circles show the results $S_e(V_e)$ using ten different statewide elections (e=1,…,10). The open circle at V = 50% is the average obtained from the $S_e(V)$ curves that are not shown for the individual elections but that are easily viewed on DRA; the standard error (0.3%) of the mean is shown by bold horizontal caps on the open circle. Also shown is the line for proportionality (dashed).

The values of P in Fig. 1 are the difference between the proportionality line and the filled circles in Fig. 2. Including the values of $S_e(V_e)$ for each statewide election e in Fig. 2 emphasizes that the values from the individual elections agree quite well with the DRA composite S(V) shown as a solid curve in Fig. 2. Each election e also generates an $S_e(V)$ curve, which generally agree well with each other and with the composite. This agreement is indicated in Fig. 2 by the open circle and its 0.3% statistical uncertainty.[18]

Responsiveness of more states

The previous example emphasizes that high responsiveness is at the root of the issue. In this section we emphasize that Virginia is more the rule than an exception. Figure 3 shows previously published (Nagle and Ramsay, 2021) S(V) curves for 11 states, here accumulated in the same graph. For all states the percentages of seats exceed proportionality for vote share V greater than 55% and for V less than 49% they are less than proportional. Ignoring interesting

[18] The differences 50 – $S_e(50)$ are the values of the seats bias α_S shown in Fig. 1 for each of the ten e elections. The average over e of $S_e(V=50)$ is 47.3% with 0.3% standard error of the mean. The size of the uncertainty in the composite S(V) curve is about the same at other V, as shown for other states by Nagle and Ramsay (2021).

differences between the states for the moment, the immediately pertinent point of Fig. 3 regarding mP is that the increase in the percentage of seats with an increase in the vote far exceeds proportionality over a broad range of vote, and importantly, this is the case for both red and blue states.

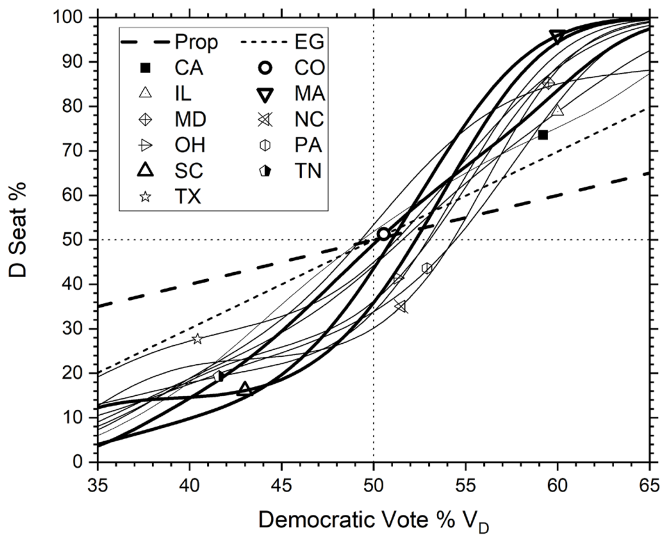


Fig. 3. The symbols show the % of Democratic seats for 2011 congressional plans using DRA composite election results and fractional seats (Nagle and Ramsay, 2021). The line passing through a state symbol is the S(V) curve generated using the proportional shift method employed in DRA. The CO, MA and SC curves are drawn bolder than the others.

To quantify the foregoing perception in a way that facilitates numerical comparisons among different states, let us define some broad responsiveness metrics. $R_{40\text{-}60}$ relates to the difference in seat percentages at 60% vote and at 40% vote and is defined as $R_{40\text{-}60}$ = (S(60)-S(40))/20 with values shown in Table 1. Also considered are two smaller ranges defined as $R_{40\text{-}50}$ = (S(50)-S(40))/10 and $R_{50\text{-}60}$ = (S(60)-S(50))/10. Only $R_{40\text{-}50}$ is shown in Table 1 as $R_{50\text{-}60}$ equals $R_{40\text{-}60}$ minus half of $R_{40\text{-}50}$. Later we will consider the narrowest responsiveness metrics obtained from the differential slope dS/dV. In Table 1 the slope evaluated at the composite vote V is designated ρ and the slope at V=50% is designed ρ_{50}. Table 1 shows results using the DRA composite elections for the 44 states that have more than one CD.

state	#CDs	D vote	$R_{40\text{-}60}$	$R_{40\text{-}50}$	ρ	ρ_{50}	SB	mP	type
HI	2	68.3	4.9	5.0	0.0	9.9	-0.1	-31.7	B
NY	26	64.8	2.2	1.7	2.2	1.7	3.6	-20.0	A
CA	52	64.2	3.2	3.9	1.5	3.5	-2.8	-19.7	A
MD	8	62.2	2.5	2.1	2.7	2.3	3.9	-16.9	A

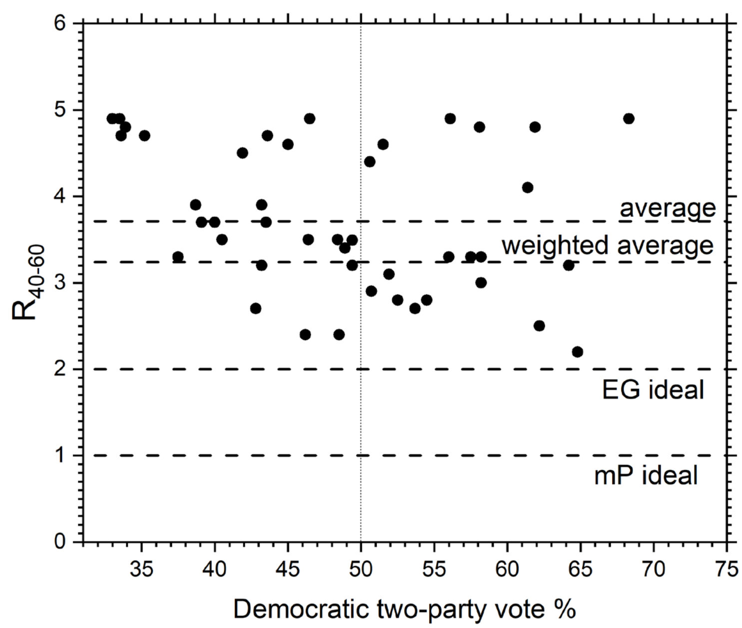
RI	2	61.9	4.8	5.0	0.7	7.1	-1.6	-37.1	B
MA	9	61.4	4.1	3.2	1.6	5.8	7.5	-34.7	B
IL	17	58.2	3.0	3.3	1.7	3.3	-3.0	-20.1	A
WA	10	58.2	3.3	3.2	3.0	3.8	7.4	-10.0	A
CT	5	58.1	4.8	4.9	2.8	7.3	-1.0	-34.5	B
OR	6	57.5	3.3	3.2	2.5	4.2	2.2	-19.7	B
NM	3	56.1	4.9	4.8	3.3	9.6	1.0	-36.2	B
NJ	12	56.0	3.3	3.7	2.7	4.5	-4.3	-18.5	B
CO	8	54.5	2.8	2.5	2.6	3.0	6.2	-2.1	A
MN	8	54.5	2.8	2.2	2.9	2.7	8.7	0.8	A
VA	11	53.7	2.7	2.3	2.8	2.3	2.3	-3.0	A
PA	17	52.5	2.8	2.9	2.7	3.1	3.2	-1.5	A
MI	13	51.9	3.1	2.7	3.5	3.5	4.5	-0.2	A
NV	4	51.5	4.6	5.8	4.2	4.3	-11.1	-16.7	B
WI	8	50.7	2.9	1.2	3.1	3.6	13.9	12.7	A
ME	2	50.6	4.4	4.4	2.8	2.7	0.9	-0.2	A
NC	14	49.4	3.2	3.1	2.6	2.7	0.3	1.2	A
AZ	9	48.9	3.4	2.2	3.5	4.3	5.9	9.1	A
GA	14	48.0	2.4	1.1	0.7	1.0	13.4	13.0	C
FL	28	48.4	3.5	2.4	2.3	3.8	7.8	10.8	A
NH	2	46.5	4.9	4.9	7.0	9.6	-0.0	26.4	B
OH	15	46.4	3.5	2.7	3.4	4.2	7.9	17.6	A
TX	38	46.2	2.4	0.7	0.5	1.2	10.8	9.8	C
IA	4	45.0	4.6	5.4	6.6	6.0	-6.2	22.4	B
MT	2	43.6	4.7	4.6	4.5	5.8	1.1	27.8	B
IN	9	43.5	3.7	1.3	0.4	5.6	17.4	21.2	BC
MS	4	43.2	3.2	0.6	0.1	0.8	19.7	18.1	C
SC	7	43.2	3.9	2.4	0.8	6.8	11.6	27.5	BC
MO	8	42.8	2.7	1.0	0.6	1.8	14.9	16.6	C
KS	4	41.9	4.5	3.7	3.1	6.1	5.1	28.5	B
KY	6	40.5	3.5	1.8	1.4	4.3	12.0	19.9	B
AL	7	40.0	3.7	2.1	0.1	6.2	14.4	25.6	BC
LA	6	39.1	3.7	1.0	0.0	3.7	23.7	22.4	C
TN	9	38.7	3.9	2.7	0.6	5.3	9.7	26.4	BC
NE	3	37.5	3.3	3.9	3.1	3.2	-9.3	25.3	A
AR	4	35.2	4.7	4.3	0.3	7.4	2.8	34.7	B
ID	2	33.9	4.8	4.8	0.0	8.3	-0.5	33.8	B
OK	5	33.6	4.7	4.6	0.1	6.7	0.2	33.5	B
WV	2	33.5	4.9	4.9	0.0	10.0	0.0	33.5	B
UT	4	33.0	4.9	4.9	0.0	12.0	0.1	33.0	B
USA	429	51.9	3.2	2.6	1.9	3.7	5.1	0.3	

Table 1. Some metrics for 2022 congressional plans of states abbreviated in the first column, with the number of congressional districts in the second column, arranged in decreasing order of the DRA 2016-2020 composite vote in the third column. Columns 4 and 5 give the values of the broad responsiveness metrics $R_{40\text{-}60}$ and $R_{40\text{-}50}$. The sixth column shows the DRA responsiveness

ρ, which is the differential dS/dV slope of the S(V) curve at the composite V. The seventh column gives the slope ρ_{50} of the S(V) curve at V=50%. The mP and SB columns give values for those two metrics. The letters in the final column roughly divide the states into three types described later in the text. The last USA row is the CD weighted average of the states.

The first important finding from Table 1 is that the broad responsiveness $R_{40\text{-}60}$ is much larger than the value of 1 that proportionality idealizes. This is shown graphically in Fig. 4. The average over all states is 3.7. This is also considerably larger than the value of 2 for the ideal efficiency gap (McGhee, 2014) and it is even larger than the value of three[19] of the classical cube law (Kendall and Stuart, 1950). Smaller states tend to have larger values of $R_{40\text{-}60}$ than more populous states but even the average weighted by the number of CDs is 3.2.[20] Figure 4 also indicates that $R_{40\text{-}60}$ is as little different for deeply red GOP states as for deeply blue Democratic states and as for more balanced purple states. The large broad $R_{40\text{-}60}$ responsiveness appears to be a feature of the single member district system independently of partisanship. Importantly, $R_{40\text{-}60}$ also appears to be independent of who draws the lines, as shown in Table 2.

Fig. 4. Broad responsiveness $R_{40\text{-}60}$ extracted using DRA for state plans as of 11/2024 for all states with more than one congressional district. Seats values used to compute $R_{40\text{-}60}$ were obtained from the DRA seats-votes graphs using its 2016-2020 composite statewide data bases.



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[19] The cube "law" has a slope of 3 at V = 0.5, but its $R_{40\text{-}60}$ is only 2.72.

[20] Goedert et al. (2024) have noted a responsiveness of 2.6 from simulations of the 34 most populous states.

Control	# states	average	# districts	average
Indy	8	3.5	107	3.2
GOP	18	3.8	176	3.3
Dems	8	3.7	75	3.1
Split	10	3.6	71	3.2

Table 2. Average $R_{40\text{-}60}$ for four types of commissions identified in DRA as independent commissions, GOP controlled, Democratic controlled and split control. Number of states and the averages by states are shown in columns 2 and 3, respectively and number of CDs and the averages by CDs are shown in columns 4 and 5, respectively.

In passing, there is also additional information in Table 1 that can be seen by examining Figure 5 which makes use of the somewhat narrower responsiveness metrics $R_{40\text{-}50}$ and $R_{50\text{-}60}$. While these values necessarily sum to $2R_{40\text{-}60}$, it is the difference that is interesting. When $R_{40\text{-}50}$ is less than $R_{50\text{-}60}$, the primary reason is that S(V=50) is smaller than 50%. This difference is closely related to the SB metric of bias, although it also depends on the value of S at V=40% and V=60%, so it can be thought of as yet another bias metric.[21] Fig. 5 shows values that indicate considerable average bias for GOP states and much less average bias for the group of Democratic states.[22]

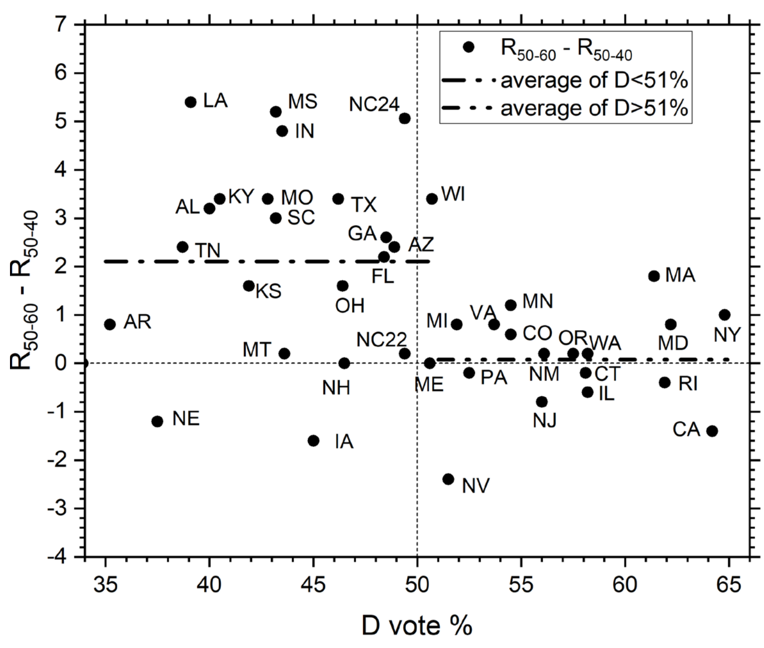

Fig. 5. $R_{50\text{-}60}$ minus $R_{50\text{-}40}$ versus statewide two party D vote.

[21] $R_{40\text{-}50}$ minus $R_{50\text{-}60}$ is also a measure of the asymmetry of the S(V) curve.

[22] Note the difference between the highly biased NC2024 and the NC2022 plans.

Some undesirable consequences of using the mP metric of bias

The most conspicuous consequence is for strongly unbalanced states. Modified proportionality (mP)[23] reports numbers for bias that tend to strongly favor the majority party. Supposing that we define an unbalanced Democratic state to have greater than 55% Democratic two-party vote, the first 12 states in Table 1 are unbalanced and mP claims they are all strongly biased in favor of Democrats. Similarly, defining an unbalanced GOP state to have greater than 55% GOP vote, the last 17 states in Table 1 are unbalanced and mP claims they are strongly biased in favor of the GOP. Importantly, this feature of the mP metric is non-partisan. Of course, any of these states may be biased, but it is likely that the mP metric exaggerates the bias because it assumes an unrealistic value of responsiveness. This is indicated by comparing to the SB values in Table 1 that are obtained using the seats metric. In HI, NY, CA, MD, RI, MA, IL, CT, OR, WA, NM and NJ the mP values are considerably more negative than the SB values. In IA, MT, SC, KS, KY, AL, TN, NE, AR, ID, OK, WV and UT the mP values are significantly more positive than the SB values.[24]

A second consequence is more important for more nearly balanced states. Plans acceptable to the mP metric are more likely to have anti-majoritarian[25] outcomes that favor one party. To see why, consider Fig. 6. This figure shows S(V) for two hypothetical plans that both have responsiveness R = 3. Let's consider what occurs when the average state vote is 53%.[26] The plan shown by the dash-dot line has the same seat % as proportionality at V = 53%, so its mP value of zero declares it to be fair. However, vote swings of 5% are normal. When the vote swings to between 50% and 52%, there is an anti-majoritarian range, in which the vote percentage exceeds 50% but the seat percentage is less than 50%. Importantly, the vote V in Fig. 6 can be either V_{DEM} or V_{GOP}, so this type of anti-majoritarianism can occur against either party. Therefore, this flaw in the mP metric is non-partisan. Table 1 indicates that states where this flaw

[23] Henceforth, mP is what DRA reported, as of 11/2024, as proportionality for its 2016-2020 composite election.

[24] However, the difference is much reduced or even slightly reversed in IN, MS, MO and LA. These exceptions come about by the combination of large seats bias and small responsiveness, as will be elaborated later.

[25] Less than half the seats for more than half the two-party vote defines anti-majoritarian.

[26] States in Table 1 with average vote for the majority party in the range 52-55% are CO, MN, PA, VA, IA, NH, OH and TX.

is most likely to occur are CO, MN, VA, PA and MI because these states all have votes that differ a few percent from 50% and values of mP and SB that have similar values as in Fig. 6.[27]

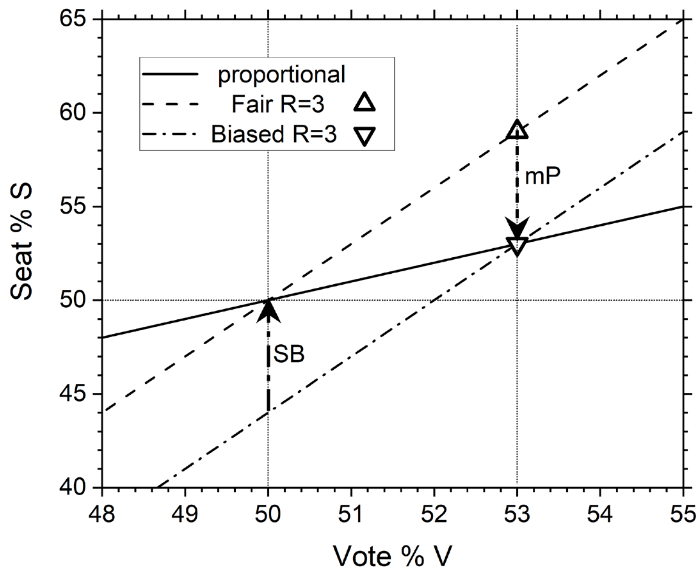

Fig. 6. Illustration of anti-majoritarianism when the mP metric claims fairness. The seats-votes curve shown by the dot-dash line is deemed fair by mP when the vote is 53%, but it becomes anti-majoritarian when the vote swings to between 50 and 52% vote, Its +6% seats bias is shown by the vertical SB arrow. The seats-votes curve shown by the dashed line has 50% seats for 50% vote but it is deemed by the mP metric to be biased by -6% as shown by the arrow labelled mP.

A third possible consequence has partisan implications. If the states with party A majority try to minimize the mP value in their plans and the states with party B majority take the larger and more natural R = 3 winner's bonus, then party A will have put itself at a disadvantage for control of Congress. For the example in Fig. 6, this disadvantage is the seats bias SB.

[27] Of course, anti-majoritarian is even more likely to occur in balanced states when mP is not small and SB is not zero. In Fig. 6 this occurs when the average minority party vote is less than 50% and the swing vote takes it to between 50% and 52%. States most likely to have anti-majoritarian tendencies favoring the GOP are WI, NC, AZ, GA, FL, OH and TX, while plans in NV and IA have Democratic anti-majoritarian tendencies.

Seats bias

Having argued that mP is not a good metric of bias, what is better? Many metrics have been proposed[28] that represent different perspectives. However, even when courts have expressed interest in measuring partisan bias, they have tended to call for a single, simple metric.[29] It was suggested that five metrics, explicitly not including mP, had special merit and, rather than choosing just one of them, that a composite of fairness metrics would provide such a single number. (Nagle and Ramsay, 2021) However, this kind of composite is not simple to calculate and some of the metrics could not be applied to plans in some states.

Here, we focus just on the time-honored seats bias SB metric. Its bedrock supporting principle is that a fair plan should make it most likely that parties that obtain equal vote shares would obtain the same number of seats. When there are two dominant parties, it is convenient to concentrate on the two-party vote share.[30] Then the fairness principle, succinctly stated, is half the seats for half the votes.[31] Deviations from this give the seats bias α_S, which is half the difference in the numbers of seats estimated for the two parties at 50% two-party vote divided by the total number of seats.[32] The historical concern with this metric has been the necessity of having to shift the vote so as to be able to draw the S(V) curve to the 50% point using data from an election where V may be far from 50%.

Importantly, the close agreement of the S(V) curves drawn from different elections (see the uncertainty noted in Fig. 2 and in (Nagle and Ramsay, 2021)), even including elections with V far from 50%, indicates that the concern with counterfactual shifting of the vote is vastly overblown. Indeed, using the suite of elections in DRA we find here that the average standard error of the mean for all 2022 state plans is 0.45% and is only greater than 1% for IL (1.27%) and KS (1.07%).

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[28] DRA reports values from other worthy metrics in its Advanced section, notably mean-median (McDonald and Best 2015), the efficiency gap (McGhee 2014,) and declination (Warrington 2018). The relatively new quadratic rule metric (Barton,2022) has merit, although it does not actually ensue from the cited appealing cut and choose districting method.

[29] Davis v Bandemer 478 U.S. 109 (1986) , at 125 n9. Justice White, Stevens and Powell concurring.

[30] It is noteworthy, however, that this principle is not limited to two parties.

[31] Plans that satisfy this principle are automatically majoritarian for typically monotonic S(V) curves.

[32] The maximum value of seats bias is only 50% due to the traditional factor of one half.

Better yet, comparison of two plans can be even more precise. For example, there was a major contender for the PA2022 plan called the GMS plan. The PA2022 enacted plan has a composite seats bias of 3.2% in Table 1 and the GMS plan has 1.9%, only 1.3% smaller. With 0.9% SEM uncertainty from Table 1, the two plans could be judged to have insignificant difference in bias within statistical uncertainty, but this ignores the correlations that occur within the suite of elections. Table 3 shows a better way to compare the two plans. The bias is compared election by election and then the differences are averaged over the suite of elections. This gives the same difference of 1.3% bias for the elections used in the DRA composite, but the standard error of the mean SEM uncertainty in this difference is only 0.24%, which means that the 1.3% difference in bias is statistically highly significant.[33,34]

Election	PA22	GMS	Δ_{13}	Δ_6
P16	4.7	2.8	1.9	1.9
S16	7.5	5.5	2.0	2.0
AG16	5.7	3.5	2.2	
T16	7.2	5.5	1.7	
Aud16	8.9	9.7	-0.8	
S18	4.8	3.3	1.5	1.5
G18	2.8	2.1	0.6	0.6
P20	0.9	0.3	0.6	0.6
AG20	0.7	-0.7	1.3	1.3
Aud20	1.3	-0.5	1.8	
T20	0.8	-0.8	1.5	
S22	0.4	-0.2	0.6	
G22	-0.2	0.2	-0.4	
Average	3.5	2.4	1.1	1.3
SEM	0.9	0.9	0.26	0.24

Table 3. Seats bias for the PA2022 enacted plan compared to the GMS plan for the statewide elections in the first column identified by year and office (P-president; S-senate; G-governor; AG-attorney general; T-treasurer; AUD-auditor). The differences are given in the Δ columns and the standard error of the means are given in the last row.

Table 3 also includes the seats bias for more elections. Arranging them by year shows that bias favoring the GOP in PA has been decreasing with time. This is largely uncorrelated with

[33] The court did not explicitly consider partisan bias but made its decision based on county splits and not splitting the city of Pittsburgh, two criteria that make it more difficult to minimize partisan bias (Nagle, 2019).

[34] A Condorcet winner may not emerge when comparing a suite of maps, in which case any of the maps in the Condorcet indeterminate set would be suitable with regard to partisan bias.

shifts in the overall vote and instead reflects shifts in voter preference between districts.[35] A refinement would be to fit the trend with time to extrapolate the bias to the time of the first election under the plan. However, that is not necessary to compare the two plans in Table 3 as both have the same temporal trend, as would be expected generally.

Further Comparisons and Types of Plans

This section will emphasize the dual importance of responsiveness in choosing good plans. Let us first focus on elections that have vote close to 50% as that has been an important way to evaluate bias (Chen and Rodden, 2013). Seven close elections are collected in Table 4. Values obtained by the proportionality metric (the P value in column 4) are unsurprisingly quite close to the seats bias SB because these two metrics are identical when the two-party vote is 50%. However, for each composite election the P value becomes mP and its values move away from the SB values, largely because the responsiveness ρ is greater than 1. In FL, AZ and NC, mP would blame the GOP for having too much bias in its favor. In the other states mP would tilt the bias toward the Democrats, erroneously attributing Democratic bias in MI, PA and VA and attributing too little GOP bias in MN. Figure 1 has already illustrated this in detail for VA.

	V	SB	mP;P	ρ
FL	48.4	7.8	10.8	2.3
S18	49.9	7.1	7.3	2.9
MI	51.9	4.5	-0.2	3.5
P16	49.9	5.9	6.2	3.3
MN	54.5	8.7	0.8	2.9
Aud22	50.2	7.1	6.9	2.2
NC	49.4	O.3	1.2	2.6
AG20	50.1	0.2	0	2.6
AZ	48.9	5.9	9.1	3.5
P20	50.2	3.9	3.5	3.8
PA	52.5	3.2	-1.5	2.7
P16	49.6	4.7	5.4	2.8

[35] There is a similar trend in some other states, but not all, but a detailed analysis of the durability of bias is beyond the scope of this paper.

VA	53.7	2.3	-3.0	2.8
AG21	49.6	2.7	3.2	2.3

Table 4. Comparison of metrics from the DRA 2016-2020 composite elections and those obtained with close elections for the 2022 congressional plans of seven states. There are two rows for each state. The first row identifies the state in the first column. Subsequent columns give the vote V, the seats bias SB, mP and responsiveness ρ at V for the DRA composite election. The second row for each state identifies the close election, P (president), G (governor), S (senate), AG (attorney general) or AU (auditor) and the year in the first column and subsequent columns give values of SB, P and responsiveness ρ at the vote V in column 2.

Another batch of states is considered in Table 5. As in Table 4, the mP values move away from the consensus range obtained from proportionality for the elections closer to 50% vote and from the seats bias for either election. However, there are two distinguishing features shared by the states in Table 5. One is that these states are unbalanced, either strongly red or strongly blue. The second is the high values of the responsiveness ρ for the election that is closest to 50%.[36] This combination raises a red flag. In a 60-40 state the majority party would obtain essentially all the seats if it drew a map with 60-40 voter preferences in all districts. This method of taking partisan advantage can be diagnosed by a high value of ρ_{50} for 50% two-party vote. However, MA is particularly interesting in this regard. Its relative geographical political homogeneity prevents drawing any district that leans to the GOP at the most likely statewide vote, so it can hardly be designated as biased against the GOP, in agreement with the rather different analysis of the similar 2012 MA plan (McDonald et al., 2018). Interestingly, the MA map has a seats bias favoring the GOP, which comes about because district MA7 is packed with 82% Dems, and seven of the nine MA districts turn red for the closest G14 election.[37] Therefore, appropriate auxiliary tests should be applied to states that are flagged by a very high responsiveness ρ_{50} at V = 50%. This includes states with high responsiveness that we did not include in Table 5 because there were no close elections. For example, the current UT map has all of its four districts with 63 to 71 % GOP preference resulting in a very high value of 12 for ρ_{50} in Table 1, but it is easy to draw a UT congressional district, entirely within Salt Lake county, that has a 55% DEM voter preference.

[36] Recall that ρ(V) is the slope of the S(V) curve right at V in contrast to the earlier defined broad responsiveness $R_{60\text{-}40}$ which is the slope from V at 40% to V at 60%.

[37] Also, ρ at the composite vote is much smaller because the districts already have a strong preference for DEM and higher DEM vote can't change them much further.

The unbalanced states with high ρ_{50} are classified as type B in the last column of Table 1. The S(V) curve for MA is shown in Fig. 7. The VA curve is also shown, but VA is classified as type A in Table 1 because it has a more normal value of responsiveness near 50% vote, which can also be seen from its value of $\rho_{50} = 2.3$ in Table 1.

	V	α_S	mP;P	ρ
CT	58.1	-1.0	-34.5	2.84
G18	51.7	- 0.9	-10.2	6.46
IA	45.0	- 6.2	22.4	6.60
G18	48.6	-6.7	0.5	6.62
MA	61.4	7.5	-34.7	1.61
G14	49.0	9.9	14.3	5.20
NM	56.1	1.0	-36.2	3.30
S20	53.1	0.6	-23.9	7.18
AL	40.0	14.4	25.6	0.10
S17	50.8	16.8	13.0	5.9

Table 5. Type B states. The format is the same as for Table 4.

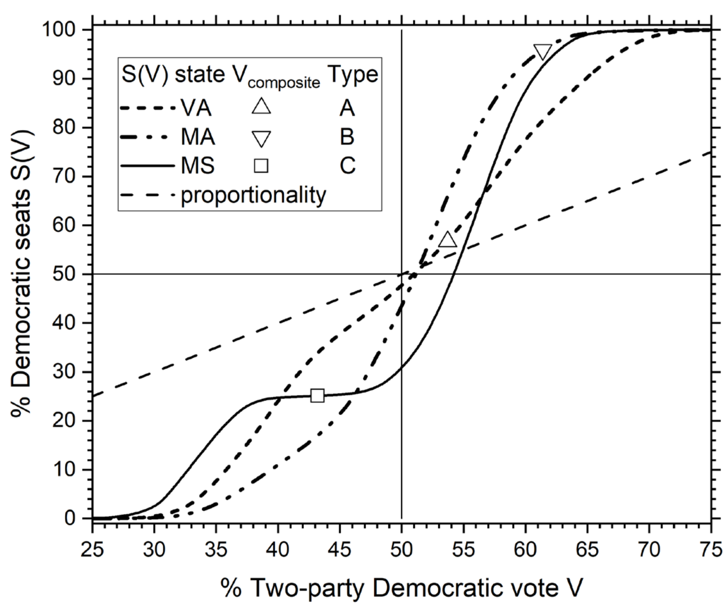

Fig. 7. Seats-votes curves for VA, MA and MS representing types A, B, and C, respectively. The open symbols show the DRA composite votes.

Another batch of states that we classify as type C is considered in Table 6. These states have small responsiveness ρ at the composite vote, but that alone does not make them type C. As shown in Figs. 3 and 7, all S(V) curves must flatten out at both high and low values of V as seats S approaches either of its extreme values of 1 or 0, and this automatically gives a small value of ρ if the composite vote places S in one of those two regions. For example, the MA composite vote places it where the S(V) is flattened near 1 in Fig. 7. In contrast, S(V) for the composite vote for MS (Mississippi) in Fig. 7 is flattened far from an extreme. We will classify states that have S(V) curves like the MS curve in Fig. 7 or the SC curve in Fig. 3 as type C. These S(V) curves look quite different from those of type A, represented by VA in Fig. 7 and by CO in Fig. 3, and from those of type B, represented by MA in Figs. 3 and 7. We now come back to footnote 24 at the beginning of the previous section, that some unbalanced states are exceptional in having a value of SB nearly the same or even a bit larger than mP. This comes about for MS in Fig. 7 because the flattening of the S(V) curve for vote between the composite vote and 50% makes the difference in seats between the composite vote and the 50% vote smaller than the vote difference. Importantly, curve flattening with small responsiveness means that the districts are safer, less competitive, for both parties, so type C maps are additionally undesirable compared to type A and type B maps that may be strongly biased, but are still responsive.[38]

	V_D	SB	mP;P	ρ
GA	48.0	13.4	13.0	0.70
P20	50.1	13.6	13.6	1.10
MS	43.2	19.7	18.1	0.06
G19	47.4	19.0	20.7	0.78
MO	42.8	14.9	16.6	0.55
S16	48.5	17.1	18.7	1.84
TX	46.2	10.8	9.8	0.54
S18	48.7	9.2	9.1	0.62
AL	40.0	14.4	25.6	0.10
S17	50.8	16.8	13.0	5.9

[38] A final possible type of S(V) curve would be both unresponsive and have small seats bias. This does not occur in any current congressional plans, but the rejected NY 2022 plan *vide infra* is such an example.

Table 6. The format is the same as for Table 3.

Results for the remainder of the states with two or more districts are only shown in Table 1 because the other elections add less additional insight to what one obtains from the composite election. These states are also classified as type B if they have values of ρ_{50} greater than 4.0. They are classified as C if their S(V) curves resemble that of MS in Fig. 7. Some states like AL and SC are classified as both B and C because they have both high ρ_{50} and low ρ. As before, values of mP and SB differ greatly for the A and B states when the vote is not close to 50%. They only become close to each other for WI, ME and NC, as they must, because the composite vote is nearly 50%. They also agree for type C plans, but that is due to the flaw of low responsiveness.

Discussion

Agreement of mP and SB in only 9 of the 44 σtates implies that they can't both be valid metrics. Of course, each state is different with its own political geography (Rodden and Weighill, 2022) within these rough groupings. One should look at other metrics not focused on in this paper and, especially, at the rank and seats figures in DRA. Nevertheless, groupings such as these may be helpful to discern broadly common features.

In particular, the separation of some of the states into types B and C emphasizes the importance of responsiveness in addition to partisan bias. The trade-off between these two criteria is illustrated by comparing metrics for the current court enabled NY plan and the rejected legislative plan that have been carefully discussed (Magleby and McDonald, 2025). Using the DRA 2016-2020 composite, the 2022 enacted plan had more seats bias, 2.9% versus 1.5% for a statistically significant difference of 0.4 seats. However, the court was more concerned with responsiveness. That was only $\rho = 1.0$ in the rejected plan, same as proportionality, whereas it increased to 2.2 in the court enabled plan, more in keeping with the broad responsiveness in Fig.4.

While responsiveness is important, we find it germane to consider three different metrics for it. The broad responsiveness $R_{40\text{-}60}$ is useful to illustrate that the USA single member district system is far more responsive than the normative proportionality ideal, whoever draws the maps.

The narrow responsiveness ρ helps diagnose unresponsive plans with mostly safe districts, the type C states in Table 1. Large values of the narrow ρ_{50} helps diagnose whether cracking voters of a minority party might be occurring in unbalanced type B states.

The last row in Table 1 averages the metrics for the congressional districts of the 44 states with more than one CD. The average $\rho = 1.9$ is smaller than the broad $R_{40\text{-}60}$ because the vote in highly unbalanced states like MA and in type C states locates them on flattened places in their S(V) curves. The average ρ_{50} is larger than $R_{40\text{-}60}$ because the S(V) curves are generally steeper in the middle. Most interesting is that mP only has 0.9% national bias whereas the national seats bias metric has 5.1%. The latter would favor a GOP average of 236.2 of these 429 seats when the national vote is 50%, to which 3.8 seats could be added for the six states with only one district, for a total of 240 GOP seats and 195 Democratic seats. The number of Democratic seats rises to 211 for their national two-party vote of 51.9% in Table 1 using the national average responsiveness $\rho = 1.9$.[39] It would appear from this analysis that Democrats would need their national average vote to increase by another 0.9% to achieve control of the House.[40] Importantly, bias estimates are not predictions of outcomes because they only take into account the voter preference of districts from statewide elections and do not take into account district incumbency and amount of campaign finances that are specific to actual elections; these should not be considered in the map drawing process even if they could be. Nevertheless, it is encouraging that the estimate of 211 DEM seats for the 2022 maps evaluated with the 2016-2020 election data is so close to the actual 2022 outcome.[41]

The introduction describes a possible normative principle behind proportionality as empowering voters of like mind equally. In the highly polarized US two party system, effective empowerment of voters of like mind is not the seat percentage divided by the voter percentage.

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[39] This is nearly identical to what Goedert (2014) reported for the previous decade.

[40] Note that this two-party national vote is district averaged. This deliberately ignores different voter turnout in different districts, quantified as turnout bias (McDonald, 2009), as irrelevant for determining seats, so this national vote is different from typical national vote tallies that would be used in National Popular Vote for president.

[41] Looking ahead to 2024, the NC2024 plan is much more biased than the NC2022 plan and there have been smaller shifts in the plans of other states with the net shift towards the GOP. There has also been a shift in seats bias with time in many states, like PA in Table 3, with the net shift toward the Democrats. Analysis has been constrained in this paper to the 2022 enacted plans and the 2016-2020 data available when those plans were approved.

Instead, actual empowerment becomes rather more like nearly complete for voters of the party in majority and closer to zero for voters in minority.[42] Therefore, there does not seem to be a valid normative principle that underlies proportionality. In contrast, the seats bias metric rests on the strong normative principle of equal seats for equal votes.

Conclusions

We have herein argued that any proportionality metric is unsuited for measuring bias in the single member district system employed in the USA. The root cause is that responsiveness in the single member district system is far from proportional, no matter who draws the lines. Consequently, the proportionality metric is not robust, varying strongly with the two-party vote. Use of just the composite election in the mP metric suppresses this problem. However, the composite vote is sensitive to the choice of which elections to include in the composite, so non-proportionality remains a problem for the mP metric.

In contrast, the traditional seats metric SB is relatively insensitive to the choice of election. Importantly, comparing many elections gives a measure of uncertainty in SB that is typically less than 1%, and the difference in bias of two plans has even smaller uncertainty when analyzed with a suite of elections. Hence, the better of two plans regarding partisan bias should usually be determinable. State redistricting commissions could use the seats metric to choose less biased plans and judges could use it to address whether an enacted plan is too biased compared to alternatives.

EndNote placeholder for footnote references. (Rodden, 2019),(McDonald and Best, 2015),(Warrington, 2018),(Keena et al., 2021),(Farrell, 2011),(Goedert et al., 2024), (Goedert, 2014), (King and Browning, 1987), (McDonald, 2009),(Barton, 2022),(Barton and Eguia, 2024)

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[42] Interestingly, such binary empowerment corresponds to the greatest satisfaction of the sum of all voters provided that the system has not led to anti-majoritarian outcomes.

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