Examples that illustrate how compactness and respect for political boundaries can lead to partisan bias when redistricting

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Abstract

To reduce the unfairness incurred by partisan districting, citizen reformers usually advocate applying the conventional criteria of compactness and respect for political boundaries of subdivisions. Simple examples are constructed to address whether optimizing these criteria also minimizes partisan bias, measured as the difference between the seats-votes curves for the two parties. One set of examples shows less bias for less compact districting, supporting the comment of Grofman and King that compactness is not a reliable proxy to achieve partisan fairness. All examples indicate that respect for political boundaries increases partisan bias, well known as unintentional gerrymandering. These examples are consistent with the contention that partisan bias should be confronted directly and minimized when redistricting.

Introduction

There is appropriate concern that single seat districts for congress, as well as other legislative bodies, have been gerrymandered. There are two pernicious effects. One is that too many safe seats have been created, members then vote to deter primary challenges rather than to represent the whole district, resulting in highly polarized legislatures. This concern is discussed using the terms competitiveness or responsiveness. Although highly worthy of concern and some of the examples in this paper will be interesting in this regard, this is not the primary focus of this paper.

The second pernicious effect, the one that is the focus of this paper, is unfair division of seats by party. This is discussed using the term partisan bias or its antithesis, the goal of partisan fairness. The concept of partisan fairness "*requires that the electoral system treat similarly-situated parties equally, so that each receives the same fraction of legislative seats for a particular vote fraction as the other party would receive if it had received the same fraction*". (Grofman and King, 2007) Traditionally, election reformers like Common Cause have been unwilling to tackle partisan fairness head on. Trying to minimize partisan bias requires using partisan data, such as previous election returns and party registration. As such data have been used to such ill effect by political parties, there is an attitude that such data should be prohibited in making redistricting maps. Instead, traditional constraints, compactness and minimizing the splits in political subdivisions, have been advocated as ways to constrain gerrymandering. However, it has been noted that "*Criteria such as compactness and respect for political boundaries are often used as proxies for partisan gerrymandering, but they are typically not very good proxies*" (Grofman and King, 2007). If this is true, then reformers should instead focus directly on partisan fairness, keeping in mind, of course, that there are other desiderata that should also be weighed into district mapping.

This paper addresses the comment of Grofman and King in the preceding paragraph using simple examples. These examples are constructed to contain the essence of issues that arise in real districting, without including many complex aspects that can easily obscure the evaluation of fundamental principles. This approach is widespread in the physical and biological sciences, where toy models are employed and cartoons are used to convey the information obtained from complex experiments. It is a valuable approach designed to promote understanding, and not one requiring apology.

Defining some basic examples

Accordingly, let us consider a simple geographical representation of a political entity. The political entity may be a county/city with county/city councils or it may be a state which has several congressional seats, as well as many state representatives and state senators; let us refer in this paper to the political entity as a state. The simplest geographical representation of the state is a circle, as shown in Fig. 1. Without loss of generality, the radius of this circle will be taken to be 1.

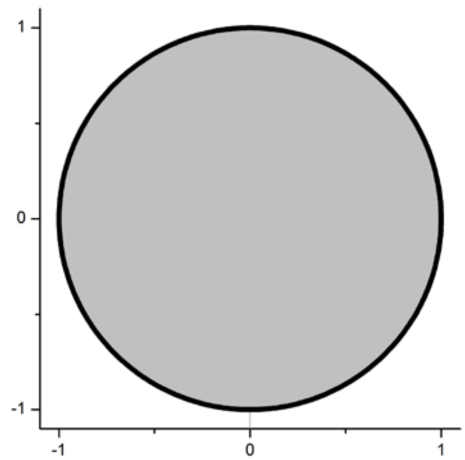


Figure. 1. A circular state with radius 1 in arbitrary units.

We will mostly consider the case when the state has to be divided into three districts. The case of two districts is not very interesting, the three district case already illustrates basic principles, and more than three districts become non-essentially more complex.

We will next assume, temporarily, that the population density is uniform in the state. Then, the fundamental principle of equal population in each district requires that all districts have the same quantitative area. Fig. 2 shows three district maps that accomplish this.

Let us now focus on compactness. We will measure compactness C by calculating the total length of the boundaries between the districts. This is similar in spirit to the (Schwartzberg, 1966) perimeter method, except that we will only apply it to all the districts at once, not to individual districts, so there is no need for complicated normalization for different size districts. The value of C for the map in Fig. 2a is the sum of the length of the three radii, each of length 1, that divide the districts. Those boundaries of the districts that are also boundaries of the state are just the circumference 2π of the circle which is the same for all district maps; they do not affect the difference in C between different maps and so are not included in our measure of compactness. Notice that the C value is substantially smaller, *i.e.* more compact, for the map in Fig. 2a than for those in Figs. 2b and 2c. It is likely that Fig. 2a is the most compact of all maps that can be drawn for a circular state with three districts, although this is difficult to prove, and such a proof is not essential for this paper. More complex measures of compactness, such as the moment of inertia method (Weaver and Hess, 1963), rank the overall compactness of different district maps similarly to the boundary method. Although it is a much more difficult calculation, it is clear that the map in Fig. 2a is more compact using the moment of inertia measure than those in Figs. 2b and 2c.

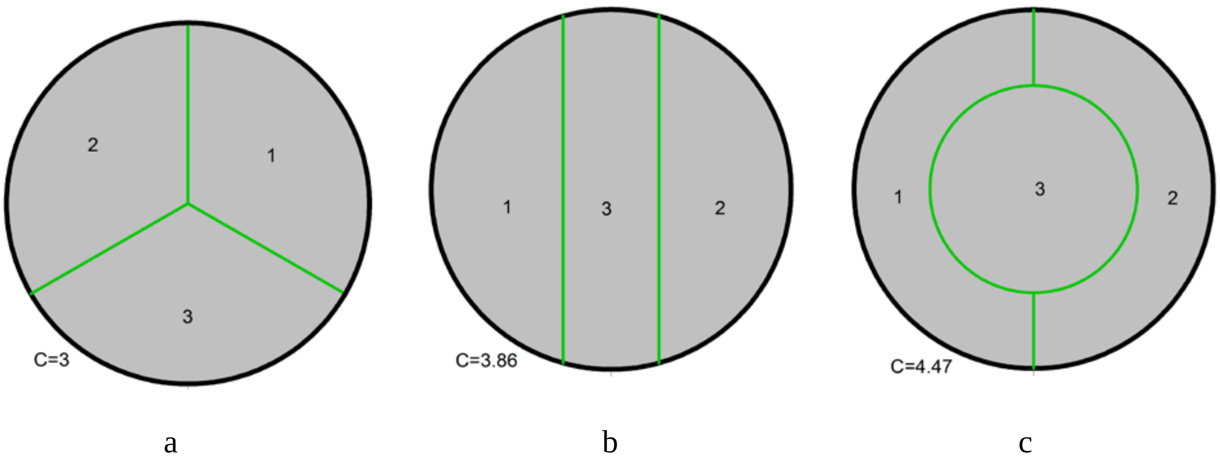
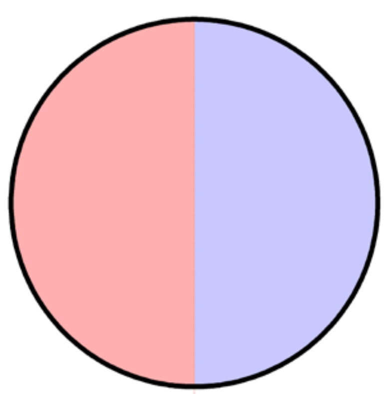


Fig. 2. Some different ways to draw three districts, labelled 1-3, for the state in Fig. 1. Compactness C is defined to be the length of the green lines that divide the state into districts.

Partisan bias is possible when different parts of the state have different fractions of partisan voters. Then voters of one party can be packed into a small number of districts while voters of the other party can be distributed into a larger number of districts with smaller, but ample majorities, leading to a larger number of seats for the latter party unless there is an improbably large swing in overall voting. Let us begin by considering the simplest possible model for how voters could be distributed geographically. As shown in Fig. 3, half the state is shaded reddish and half is shaded bluish. Let us suppose, for the moment, that 35 percent of the voters in the reddish half of the state lean blue and 65% lean red, whereas in the bluish half of the state, 65% lean blue and 35% lean red. In this special case, half the state overall could be predicted to vote red and half blue.

Fig. 3. Partisan preferences for first set of examples.



However, the overall percentage of partisan votes will generally vary from election to election and be different from 50% Let us define P to be the percentage of blue votes, and Q=100-P to be the percentage of red votes, the special case so far being P=Q=50% . If P becomes 55% in an election, *i.e.*, a shift of 5%, then the percentage P of blue votes in each part of the state would be expected to increase. Let us suppose that 70% of voters in the bluish half of the state and 40% in the reddish half then vote blue; this is the uniform shift assumption. The uniform shift assumption fails for large shifts and it has been improved upon, especially when more detailed information is applied (Gelman and King, 1994), but it is quite adequate for the examples in this paper and it avoids complicating the calculations.

We now address partisan bias for the population map introduced in the previous paragraph (Fig. 3) and for the district map in Fig. 2a. The combination is shown in Fig. 4a. District 1 is solidly reddish until the overall blue vote P exceeds 65% and district 2 is solidly bluish until the overall red vote Q exceeds 65%, while district 3 is more competitive changing from red to blue when P reaches 50%. This is quantified in the seats-votes curve in Fig. 4b which shows zero blue seats for P smaller than 35%, one blue seat for 35%<P<50%, two blue seats for 50%<P<65% and three blue seats for P greater than 65%. (The uniform shift assumption prevents P becoming smaller than 15% or larger than 85%, unlikely shifts in American politics where swings greater than 5% are rare.) This example, which we will call number 1, clearly has no partisan bias, as both parties have equal expectations upon shifting of the overall votes P (and Q) by equal amounts in opposite directions. This is further emphasized in Fig. 4b by plotting red seats versus red votes Q and obtaining the same seats-votes curve as blue seats versus blue votes, so the curve shown is colored purple for blue plus red.

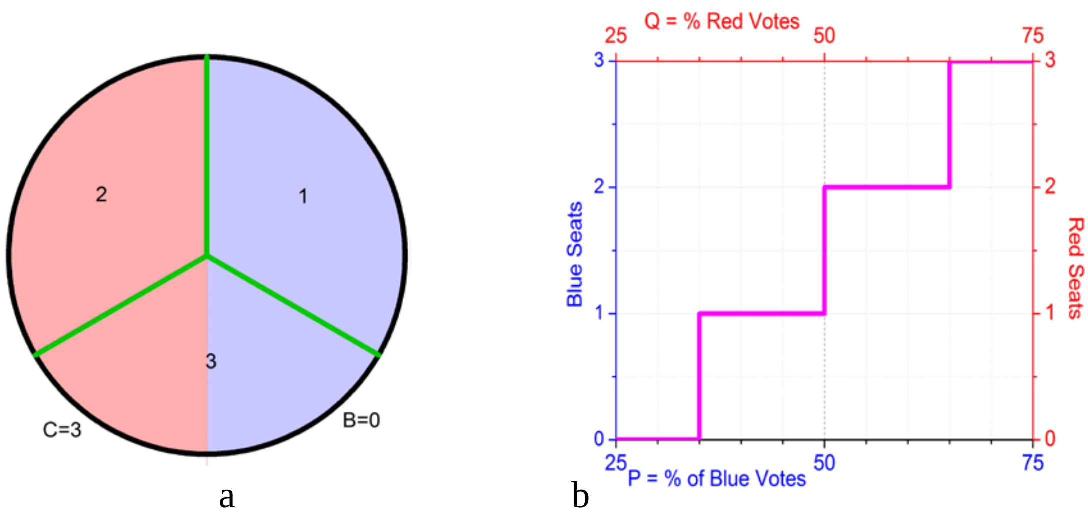


Fig. 4. Addressing partisan bias for example 1. a) Districts in Fig. 2a superimposed on the partisan preferences in Fig. 3. b) The purple line is the seats-votes curve. It is the same for blue seats (left axis) versus percentage blue votes P (bottom axis) as for red seats (right axis) versus percentage red votes Q=100-P (upper axis). The parties are treated symmetrically and partisan bias B is zero.

It is easy to see how a partisan gerrymander can upset the partisan fairness of example 1 even while completely satisfying the compactness criterion. Fig. 5a shows the same population map as in Fig. 3, but the district map has been rotated. One district, number 1, remains solidly blue but both districts 2 and 3 elect red seats until the overall blue vote P exceeds 57.5% as shown by the blue seats-votes curve in Fig. 5b. If the overall state vote is typically near P=50%, this map provides the red party with 2 seats most of the time. The outcome is clearly different as P shifts in opposite directions.

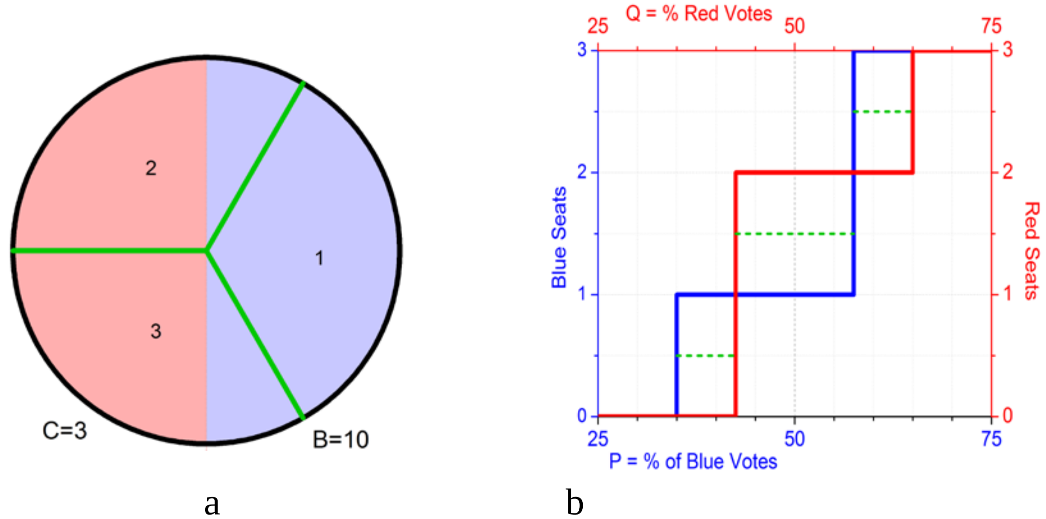


Fig. 5. Addressing partisan bias for example 2. a) The districts in Fig. 2a are rotated relative to the partisan preferences in Fig. 3. b) The blue line is the total number of blue seats (left axis) versus blue votes (bottom axis). The solid red line is the total number of red seats (right hand axis) versus red votes (top axis) plot. The measure of bias, B=10, is 1/3 the sum of the lengths of the horizontal dashed green lines between the two curves; this is also 1/3 the sum of the areas between the red and blue lines.

There is clearly partisan bias in the district map in Fig. 5a. This is often identified as asymmetry in the seats-votes curve as shown in Fig. 5b. Although the simplest indication of partisan bias is the outcome that one party receives more than half the seats with less than half the votes, such outcomes can occur due to statistical fluctuations, even with no partisan bias. This test also does not help when a party receives more than half the votes and many more than half the seats; that test is also conflated with the issue of competitiveness. Partisan bias in the map in Fig. 5a is apparent as asymmetry in the seats vote curve in Fig. 5b, at least for those who can see such things in a graph, something that judges may have difficulty with. (Mathematically, the red curve is the inversion of the blue curve through the point, f=1/2, S=3/2, located in the middle of the graph, and differences mean that the curve is not symmetric with respect to inversion symmetry.) For comparing partisan bias in district maps, a single partisan bias number is needed, just as it is needed for evaluating compactness.

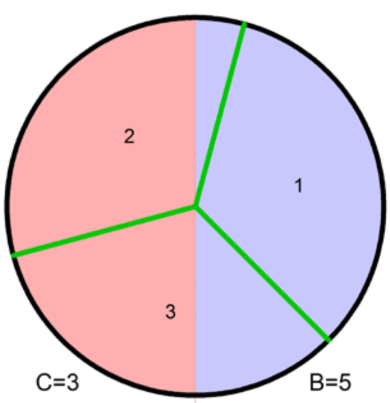
Simple quantitative definition of partisan bias

We define partisan bias B by measuring the difference between the blue seats - blue votes curve and the red seats – red votes curves. Recall that the blue seats-votes curve plots the number of blue seats S_{blue} versus the percentage P of statewide blue votes and the red seats-votes curve plots the number of red seats S_{red} versus the percentage Q=100-P of red votes. Fig. 5b shows the two curves for the map in Fig. 5a. For example 1 in Fig. 4a, Fig. 4b shows that these two seats-votes curves are identical (the curve in Fig. 4b is colored purple for red plus blue). As there is no difference between the two curves, our measure of partisan bias obtains B=0, the obviously appropriate number. In contrast, for example 2 in Fig. 5 the difference between the blue and red curves is not zero for any number of seats from 0 to 3. A simple quantitative measure of bias is the sum of those absolute differences; in Fig. 5b this is the sum of the absolute lengths of the dashed green lines. (Note that the signed differences for red minus blue necessarily sum to zero.) This sum also is the sum of the areas contained between the blue and red lines. Finally, to compare bias for different states with different numbers of districts, it is appropriate to calculate an average bias B by dividing the sum by the total number of seats, three in these examples.

More about examples 1 and 2

Examples 1 and 2 can be used to address how political subdivisions can lead to more partisan bias. In the simplest case, suppose there are also three subdivisions and their boundaries are aligned as in the map in Fig. 5a. That map would then not split political subdivisions whereas the map in Fig. 4a would split each political subdivision once. In this case, respect for political boundaries is worse than being an inadequate proxy for fairness, it is inimical to it. The term "unintentional gerrymandering" is often used to describe the packing of city voters into a few highly partisan districts, like district 3 in Fig. 5a. Although such packing is not necessarily classic "gerrymandering", as the districts can be quite compact, it emphasizes the unintended consequences of respecting political subdivisions.

The difference between Fig. 4a and Fig. 5a is the rotation of the districts by 30 degrees. For completeness, let us consider other rotations. Fig. 6a shows example 3 which has a rotation of the districts by 15 degrees, half as much as example 2. Fig. 6b shows the seats-votes curves from which a bias half as much as example 2 is obtained. Generally, the bias is linearly proportional to the rotation angle. If no other criteria, like political boundaries or partisan fairness or competitiveness, are used to determine the rotation angle, then its value would be a matter of chance and the probable value for partisan bias is the value B=5 obtained by averaging over all rotations. This is the value that we will assign to the compact district map of Fig. 2a.



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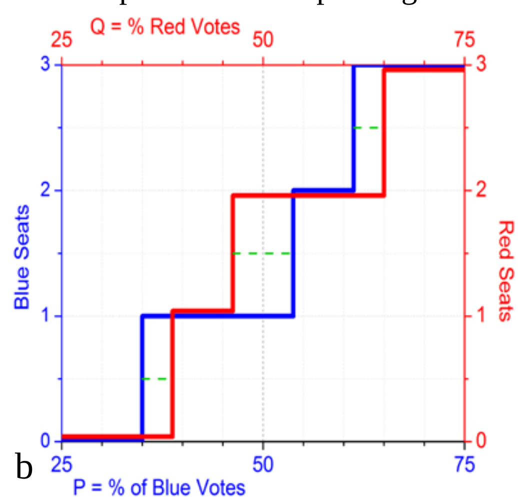


b

Fig. 6. Example 3. District rotation is halfway between Examples 1 and 2. Bias is also halfway between.

Examples showing that less compact districts can lead to less partisan bias.

Let us now consider the partisan bias for the less compact districts in Figs. 2b and 2c when applied to the partisan preferences in Fig. 3. Figure 7a shows the original orientation of the districts; it has one competitive district and two safe districts, when the overall percentage of votes is in the vicinity of 50%. Fig. 7b shows a rotation that has three competitive districts. Figure 7c shows the seats votes curves for both these rotations and also for an intermediate one that is rotated by 45 degrees from either of those in Fig. 7a or 7b. The red and blue seats curves are identical for any rotation, so this way of slicing the districts results in partisan fairness, B=0.

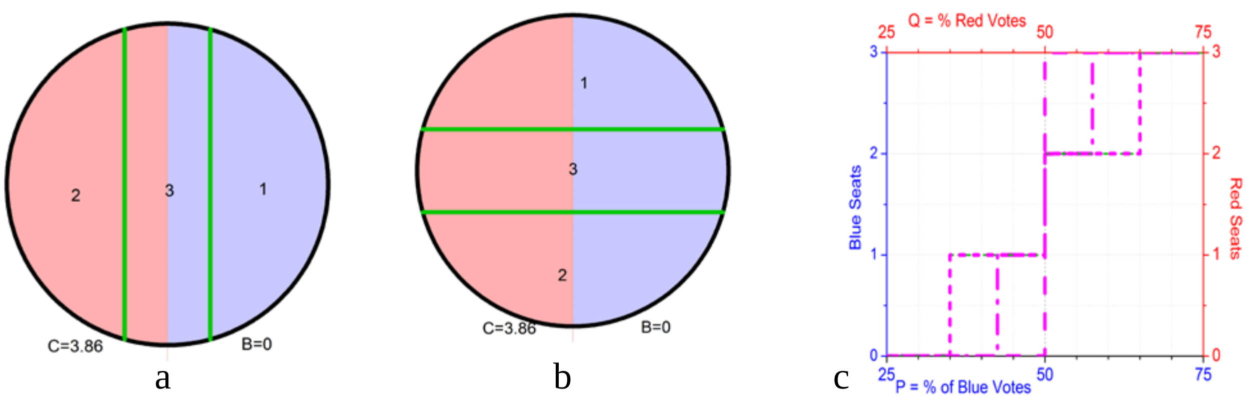


Fig. 7. Examples 4: a) and b) Two rotations of districts in Fig. 2b superimposed on the partisan preferences in Fig. 3. c) The purple lines are the seats-votes curves for three rotation angles, $\theta=0$ (short dashes), $\theta=45^{o}$ (dash-dots), $\theta=90^{o}$ (long dashes). Each curve is the same for blue seats (left axis) versus percentage blue votes P (bottom axis) as for red seats (right axis) versus percentage red votes Q=100-P (upper axis). The parties are treated symmetrically and partisan bias B is zero.

A major conclusion follows by comparing examples 1-3 with examples 4. Both sets of examples have the same partisan voter preferences. Any way of rotating the districts in Fig. 7 results in zero partisan bias, less than the average partisan bias B=5 for the earlier set of samples. In contrast, the earlier set is more compact, C=3, compared to the districts in Fig. 7 that are less compact with C=3.86. This shows that drawing more compact districts does not necessarily lead to partisan fairness; indeed, it shows that compactness can be inimical to partisan fairness.

It is true that one can achieve the greatest compactness (minimal C) and zero compactness with the same map, namely, the one in Fig. 4a. However, this would require picking one special rotation angle out of the haystack of rotation angles. Of course, this can be done, but only by using partisan voter preferences and deciding that minimizing partisan bias is a goal. Without that information, one should expect partisan bias for the most compact districts. With partisan preference information, an unbiased districting commission could achieve zero bias, and a biased districting commission could achieve maximal bias as in Fig. 5. It is therefore necessary that partisan voter preference information be allowed in redistricting and that minimizing partisan bias be a justiciable principle.

More about the previous example 4 and noting example 5

As already noted, example 4 also is interesting for the issue of competitiveness. As the angle of rotation increases from 0 to 90^o, the two uncompetitive districts at $\theta=0$ become more competitive until all three districts become equally competitive at $\theta=90^o$ and the seats-votes curves are the same as for winner takes all elections. Even though all three seats-votes curves in Fig. 7c have zero bias, there is still room for partisan gain that arises when the average overall voter partisan preference is different from 50%. For example, if the historical average vote is 55% blue, then it is a blue partisan advantage to choose the districts as in Fig. 7b because then there will usually be three blue seats. Whereas, if the districts were chosen as in Fig. 7a, there would usually only be two blue seats.

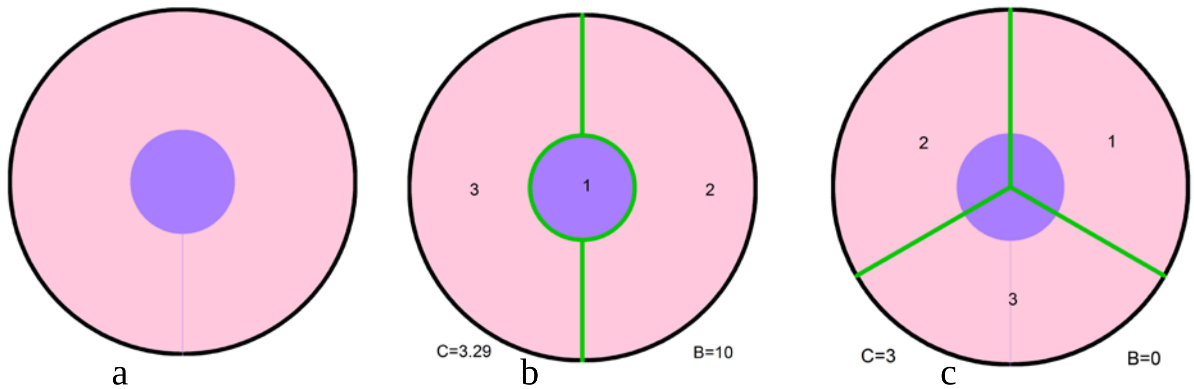
We define example 5 to use the districts in Fig. 2c with the partisan voter preferences in Fig. 3. This example is very similar to example 4, so figures will not be presented. Again, there are different rotation angles. Conveniently, the same seats-votes curves are obtained as in Fig. 7c. Bias is always zero and the competitiveness varies in the same way with the rotation angle.

Examples with different partisan voter preferences

A different partisan voter preference model is shown in Fig. 8a. There is a central bluish circle that emulates a central city in the geographically larger state that is colored reddish outside the central city circle. The model assumes that 1/3 of the people live in the central city and 2/3 live in the suburbs and outlying areas. As cities are more densely populated, the model assigns a population density five times greater than for the non-city and this means that the geographical area of the city is only 10% of the non-city geographical area. Finally, when the overall vote is 50%, the model assigns the blue voter preference to be 65% in the city and 42.5% for the non-city area.

Let us consider the two district maps shown in Figs. 8b and 8c, superimposed on the voter preference map. The map in Fig. 8b respects political subdivision boundaries by packing the city votes into one central district. The seats-votes curve for this map is identical to Fig. 5b that has the maximum bias B=10. The map in Fig. 8c cuts the city into three pieces and balances each of those bluish pieces with reddish non-city pieces. The seats-votes curve for this map is identical to the winner-take all long dashed curve in Fig. 7c. It has zero bias B and it is totally competitive/responsive near 50% overall vote. The map in Fig. 8c is also more compact than the map in Fig. 8b.

The comparison of the maps in Fig. 8b and 8c illustrates how respect for political subdivision boundaries is not just a poor proxy for partisan fairness. It even prevents partisan fairness. This is often referred to as “unintentional gerrymandering”, but it isn’t really gerrymandering in the original historical sense of wildly drawn district lines. Rather, it is simply that packing of city voters results in partisan bias.



a b c

Fig. 8. a) Voter preference map for a central bluish city with high population density embedded in a reddish non-city state. b) three equal population districts that have the same seats-votes curve as in Fig. 5b. c) three equal population districts that have the same seats-votes curve as the long dashed curve in Fig. 7c.

These two examples illustrate another important point. It is sometimes supposed that achieving partisan fairness requires drawing non-compact districts. However, in these examples, it is the map in Fig. 8c that is more compact. Unlike the earlier examples that used Fig. 3 for partisan preferences, in the present example minimizing compactness leads to smaller partisan bias. However, the difference in the values of C for the two maps is rather small, consistent with the overall conclusion that it is not a very strong proxy. Finally, these examples in Fig. 8 illustrate that the goals of compactness and respect for political boundaries need not be satisfied by the same map.

Discussion and Conclusions

The simple examples in this paper support the comment in the introduction of (Grofman and King, 2007) that compactness and respect for political boundaries are not necessarily good proxies for partisan fairness. The first pair of examples in Figs. 4 and 5 emphasizes how respect for political boundaries can lead to partisan bias. This conclusion is not so surprising. It is more surprising that the less compact districts shown in the models in either Fig. 2b or 2c have less partisan bias as shown in Fig. 7 than the more compact districts in Fig. 2a when the latter districts are averaged over random rotations as in Figs. 4-6. This confirms that compactness can lead to partisan bias. These examples all used the partisan preference map shown in Fig. 3. A different, and arguably more realistic, partisan preference map is shown in Fig. 8a. Again the district map in Fig. 8b that respects political boundaries leads to greater partisan bias than the map in Fig. 8c that pairs city and suburban voters. It is especially important that the map in Fig. 8c is also more compact, indicating that achieving partisan fairness does not necessarily require that compactness be sacrificed, as well as that compactness and respect for political boundaries can be antagonistic criteria.

The examples in this paper lead to the not surprising conclusion that there is tension between satisfying the three different redistricting criteria focused on in this paper, namely, partisan fairness, compactness, and respect for political boundaries. An easy way to resolve such tension is to ignore two of the criteria. Given that the legislature and the courts apparently can’t see non-compactness when it stares them in the face, and given that the courts and even idealistic citizen groups appear to be reluctant to actively promote partisan fairness, respect for political boundaries (and communities of interest) would be the default criterion used for districting. However, the examples in this paper indicate that would likely to lead to the most partisan bias. That seems unfortunate. It seems, instead, that the criteria should first be re-evaluated and rank ordered as to what is fundamentally most important rather than what appears to be incrementally achievable.

Let me therefore make a few brief remarks about the fundamental merits of the three criteria. Political boundaries have often come about for obscure reasons that are no longer relevant. They are often regionally pernicious with regard to cities and suburbs, creating unfair tax structures, such as the inability of cities to adequately tax workers who reside outside the city. Accordingly, perpetuating the effect of such political boundaries when drawing of congressional districts is a dubious criterion. Compactness has been strongly criticized as being ill defined (Young, 1988), although progress has been made (Chou et al., 2013). Nevertheless, compactness, by itself, merely leads to prettier maps. The hope of reformers is that compactness will indirectly constrain political redistricting chicanery REFS. However, this paper agrees with (Grofman and King, 2007) that compactness may not do much for what seems obviously most important. Partisan rivalry resonates with American culture that places great value on head to head competition, provided that the rivalry is fair and unbiased. Although bias inevitably occurs when trying to balance many different principles REFS, fairness should remain a core principle worth striving for. As for the relative standing of the three criteria addressed in this paper, I agree with the paper of (Hirsh and Ortiz, 2005) that rank-ordered respecting political boundaries and compactness behind “Promote partisan fairness and competitiveness”.

Recognizing that partisan fairness and competitiveness are actually two different criteria (King and Browning, 1987), we have focused on partisan bias. Although competitiveness has not been featured, it is interesting that it could be so radically altered by rotating the districts boundaries in Fig. 6 and 7. One problem that courts appear to have with the concept of partisan bias is whether it is justiciable (Grofman and King, 2007) and this, like the problem with compactness and competitiveness, is connected to how to measure it and how much is too much? This paper has introduced a particularly simple, quantitative measure of partisan bias that gives a single number B, so we are in agreement with (Grofman and King, 2007) that partisan bias is easily measurable. However, we defer the important issue of comparing our measure with other measures of bias.

Finally, toy models of the sort constructed in this paper are invaluable for illustrating principles in the simplest possible terms. Of course, estimating how much compactness and political boundaries alleviate partisan bias in the real world requires leaving the realm of the abstract. What is needed is to compare the actual outcome of an obviously gerrymandered state with the probable outcomes from alternative maps created to test the efficacy of the various criteria. One such map could maximize compactness, another could minimize splitting political boundaries, another could attempt to minimize partisan bias directly, another could maximize competitiveness, and another could focus on communities of interest. Together with outcomes from maps created using various combinations of weighted criteria, correlations between the various criteria would likely emerge. Such information should be important for redistricting reformers.

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Bibliography

CHOU, C. L., KIMBROUGH, S. O., MURPHY, F. H., SULLIVAN-FEDOCK, J. & WOODARD, C. J. 2013. On empirical validation of compactness measures for electoral redistricting and its significance for application of models in the social sciences *Social Science Computer Review,* 00(0)**,** 1-10.

GELMAN, A. & KING, G. 1994. A Unified Method of Evaluating Electoral Systems and Redistricting Plans. *American Journal of Political Science,* 38**,** 514-554.

GROFMAN, B. & KING, G. 2007. The future of partisan symmetry as a judicial test for partisan gerrymandering after LULAC v. Perry. *Election Law Journal,* 6**,** 2-35.

HIRSH, S. & ORTIZ, D. 2005. Beyond Party Lines: Principles for Redistricting Reform.

KING, G. & BROWNING, R. X. 1987. Democratic Representation and Partisan Bias in Congressional Elections. *American Political Science Review,* 81**,** 1251-1273.

SCHWARTZBERG, J. E. 1966. Reapportionment, Gerrymanders, and the Notion of Compactness. *Minnesota Law Review,* 50**,** 443-452.

WEAVER, J. B. & HESS, S. W. 1963. A Procedure for Nonpartisan Districting - Development of Computer Techniques. *Yale Law Journal,* 73**,** 288-308.

YOUNG, H. P. 1988. Measuring the Compactness of Legislative Districts. *Legislative Studies Quarterly,* 13**,** 105-115.