Following is a description of how I evaluated political metrics for the N8 plan. Consider the following spreadsheet:



All numbers are retained to high precision – only the display truncates for easier reading. The rows are rank ordered by partisan preference.

This analysis aggregated all statewide election results from 2012 and 2014 and registration at the precinct level. The %D vote was the two party D vote for each district. The average or mean of the two party D vote over districts was 54.87%. To obtain the votes focused mM measure, find the median vote. For an even number of districts, this is the average vote of the 9th and 10th ranked districts; in the table this is (54.2+51.7)/2. Subtracting this from the mean 54.87 gives mM = 1.92%. Note that I have calculated the Mean minus the median using a ranked list whereas the literature often refers to this as the median minus the Mean in reference to a seats/votes curve; this difference merely changes the sign in the calculation to give the same bias. For this N8 example, the bias clearly favors Republicans because, even with 54.87% of the vote, they are not doing as well in the competition for the median seat as their average vote would lead one to expect. Another way to see this is that If the average vote dropped to 50%, the median seat would be won by the Republicans, giving them more than half the seats. Clearly, mM is a simple metric to evaluate, which is why it has been gaining popularity.

To obtain a seats measure at 50% vote, the %D district votes have to be shifted. The first D Flips column in the table above used the uniform shift method. This assumes that the same 4.87% of voters for Democrats switch to vote for Republicans in each district. This trivial step is not shown in the table. What is shown in column D\Flips\uniform\shift is (100-%D+4.87)/100; this is the fractional statewide D vote at which a district is predicted to flip from R to D using the uniform shift. The uniform shift becomes progressively less realistic the further the district vote differs from 0.5. I have a more realistic way to perform a shift which I advanced in my first paper in Election Law Journal and which I am now calling the proportional shift because it assigns the same probability that a voter shifts regardless of which district the voter lives in. Both shifts are shown in the following figure. The take home here is that the two shifts agree for the crucial districts that have vote near 0.5, so it doesn’t make much difference for present purposes which shift one uses. This figure is also what I have been calling the Seats/Votes curve in my published papers; I am now calling it a Flip graph for reasons that I explain below.



There is a straightforward, but ultimately naïve, way to use this figure to estimate the number of D seats when the two-party D vote is 50%. Simply to count all those districts that flip to D with less than 50% statewide D vote on the horizontal axis. For this figure that gives 7 D seats. However, the districts with Flip vote near 50% are very competitive. Obviously, as is well recognized in the literature, the probability of those districts electing a D is closer to 50% than to 100%. I estimate that probability for the districts in the plan in the column labelled D seat\prob\sum\7.48 using a probit function with variance 0.04. (This value of the variance means that the statewide vote is estimated to be between 0.46 and 0.54 with probability 0.68269.) The numbers 7.48 is the sum of the probabilities of D wins over all the districts. This value doesn’t change much for variances that are realistic for American elections.

The probit values for district probabilities can also be used to estimate the number of responsive seats. In the final column of the table I calculated 1-4(seat prob - 0.5)2 which uses the second moment of the probability distribution. Summing over districts evaluates the number of competitive districts as 7.01. Of course, the simplest way to count responsive districts is just to assume a cutoff value, like all districts with vote between 45% and 55%. The table also shows this to be 7 districts for N8, but the two ways of calculating responsive districts don’t always agree so well for all plans.

Summarizing so far- it is trivial to apply a uniform shift to obtain values of bias and competitiveness as shown in the table above and not much harder to do it for my proportional shift, but the two methods agree quite well for evaluations of bias and responsiveness typically differs by less than 0.3 districts. As discussed in my first ELJ paper, my proportional method shifts voters proportionally no matter which district a plan puts them in, which is obviously better. Thereby, it also avoids the obvious fault that a uniform shift can result in districts with partisan preference less than zero or greater than 1.

So here is the slightly more complicated proportional shift formula which is described in the appendix to my first ELJ paper. The table below shows the results. For the 12 districts with %D greater than 50%, the D\Flips\prop\shift column shows 54.87/(0.02\*%Dcolumn) and for the 6 districts with %D less than 50% the D\Flips\prop\shift column shows 100(1-(1-.5487)/(.02\*(100-%D)). Although this proportional shift requires two steps in spreadsheet calculations, that is trivial to program.



As I mentioned above, the Flip graphs are not truly seats/votes curves. The only point on the seats/votes curve that the above calculations give is seats for 50% statewide vote. However, I have written a Fortran computer program that uses the above concepts to calculate the expected number of seats for any statewide vote. I haven’t published any of the corresponding figures yet. The one below compares the flip data (scatter points) with the continuous S/V curves. As one would expect, the S/V curves are smoother than the flip data. See MeasuringRedistrictingBias&Responsiveness.pdf for update.

